

BUSINESS ARITHMETIC

BY

C. M. BOOKMAN

FORMERLY HEAD OF MATHEMATICS DEPARTMENT

HIGH SCHOOL OF COMMERCE

COLUMBUS, OHIO



AMERICAN BOOK COMPANY

NEW YORK

CINCINNATI

CHICAGO

COPYRIGHT, 1914, BY

C. M. BOOKMAN.

COPYRIGHT, 1914, IN GREAT BRITAIN.

BOOKMAN'S BUS. ARITH.

E. P. 6

PREFACE

STRICT conformity to modern conditions is the keynote of this Business Arithmetic. Through the elimination of useless material the complete subject is presented in a one-term course.

Speed, accuracy, correct methods, — these are the demands of modern business and they have been fully met. Abundant oral and written problems, and vocational, agricultural, and manual training applications have been provided.

Profit is the ultimate goal of all business. Yet with all its systematizing, modern business, until recently, figured its profit on the cost of an article instead of on its selling price.

The author believes that the direct methods of presentation, the rapid calculation methods, the constant application of aliquot parts, the use of the equation in percentage calculations, the combining of rules and explanations, and the recognition of the latest method of figuring profits make of the Business Arithmetic a distinctly valuable equipment which business men will not fail to discover and commend.

Acknowledgments are due to I. R. Garbutt, of Cincinnati; H. T. McMyler, of Cleveland; F. H. Hamm and W. C. Humpton, of Columbus, Ohio, for valuable material and criticisms; and to Treman, King & Co., of Ithaca, N. Y., for the Retail Merchants' Table.

TO THE PUPIL

KEEP these two things in mind : It is *necessary* to be *accurate* ; it is *essential* to be *rapid*. Neither accuracy nor speed can be acquired without practice. First learn the method, then drill and drill until you *know* that your work is right. As you gain confidence in yourself you will work faster.

The exercises, drills, and problems in this book are worth to you all the effort you will put on them ; they will not waste your time on what you already know. Work out the exercises, do the drill work, think out the problems. There are no tricks or puzzles to confuse you, and the problems are no harder than those you will have to solve later. Then, when you enter business, you may know that you are better fitted for it than the average person.

Above all, don't be satisfied to do only *fairly* well what ought to be done *very* well. Some one is going to do it better than any one else ; why not you ?

CONTENTS

	PAGE
ADDITION	7
SUBTRACTION	17
MULTIPLICATION	23
DIVISION	28
MISCELLANEOUS PROBLEMS	30
PROPERTIES OF NUMBERS	38
FRACTIONS	43
Reduction of Fractions	44
Aliquot Parts	47
Addition of Fractions	51
Subtraction of Fractions	52
Multiplication of Fractions	53
Division of Fractions	58
DENOMINATE NUMBERS	63
Reduction of Denominate Numbers	73
Addition and Subtraction of Denominate Numbers	73
Multiplication and Division of Denominate Numbers	74
Denominate Numbers by Aliquot Parts	77
GRAPHS	79
MISCELLANEOUS PROBLEMS	83
THE EQUATION	88
MENSURATION	92
PRACTICAL MEASUREMENTS	100
Special Methods	106
Differences in Time	106
Approximations used in Business	107
Application of Practical Measurements to Manual Training	107
MISCELLANEOUS PROBLEMS	109

	PAGE
PERCENTAGE	117
Gain and Loss	120
Commercial Discount	123
Marking Goods	128
Wholesale and Retail Profits	130
Commission	133
MISCELLANEOUS PROBLEMS	137
INTEREST	144
Common Interest	144
Exact Interest	151
Compound Interest	153
Sinking Funds	155
BANKS AND BANKING	157
Bank Discount	164
Present Worth and True Discount	168
Partial Payments	169
MISCELLANEOUS PROBLEMS	172
TAXES	180
Apportionment of Taxes	181
Indirect Taxes	183
INSURANCE	185
Fire Insurance	185
Life Insurance	189
STOCKS AND BONDS	192
EXCHANGE	201
Domestic Exchange	202
Foreign Exchange	209
PARTNERSHIP	213
RAILROAD RATES	215
PARCEL POST	219
MISCELLANEOUS PROBLEMS	221
REVIEW PROBLEMS	227
APPENDIX	237
The Metric System	237
Values of Foreign Coins	241
Square Root	242

BUSINESS ARITHMETIC

ADDITION

1. Two things are important in addition : first, the figures must be plain ; second, they must be arranged in straight columns or rows. Carelessness regarding these two details is the cause of many mistakes.

DICTATION WORK. Write the following exercises from dictation, making the figures plain and arranging them in straight rows and columns, as for addition :

320615	487931
826704	593180
114329	607852
275016	738548

2. The basis of rapid addition is the “grouping” of figures. By practice the eye can be trained to group or combine two, three, or four figures into sums which are recognized at sight, just as we recognize words at a glance without spelling out the letters.

3. Two-figure combinations. The following drill, exercises 1 to 5, contains all the possible groups or combinations of two figures each except 1 and 0, 1 and 1. In adding them, *think results*, not separate numbers. Thus (first columns), think 9, 11, 7, 4, 10 ; not 1 and 8 are 9, 7 and 4 are 11, etc.

Give results from left to right ; from right to left ; from top to bottom ; from bottom to top ; at random.

9

7. Drill on the following groups, using two-figure combinations and combinations of 10; the 60 combinations should be read in one and one half minutes or less.

1.	1	7	9	4	6	3	8	4	9	5	8	9	6	4	2
	4	6	3	8	6	9	5	6	6	8	7	9	5	6	8
	<u>3</u>	<u>9</u>	<u>4</u>	<u>7</u>	<u>7</u>	<u>8</u>	<u>5</u>	<u>5</u>	<u>8</u>	<u>7</u>	<u>5</u>	<u>6</u>	<u>5</u>	<u>7</u>	<u>9</u>
2.	7	8	8	6	6	7	5	2	2	6	7	8	6	7	6
	4	4	6	6	4	7	7	2	3	2	1	1	4	7	6
	<u>2</u>	<u>4</u>	<u>1</u>	<u>4</u>	<u>4</u>	<u>6</u>	<u>3</u>	<u>8</u>	<u>7</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>4</u>	<u>5</u>
3.	9	7	8	8	5	3	1	7	3	9	3	5	2	3	5
	8	2	4	7	9	8	3	2	8	7	8	8	7	8	4
	<u>2</u>	<u>2</u>	<u>2</u>	<u>5</u>	<u>6</u>	<u>9</u>	<u>2</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>4</u>	<u>7</u>	<u>1</u>	<u>6</u>	<u>6</u>
4.	1	2	9	8	2	2	7	3	8	8	7	9	9	3	3
	4	9	9	8	8	4	8	6	9	5	6	5	6	4	3
	<u>6</u>	<u>9</u>	<u>9</u>	<u>4</u>	<u>6</u>	<u>4</u>	<u>5</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>8</u>	<u>2</u>	<u>1</u>	<u>7</u>	<u>9</u>

8. Add the groups in rows 1 and 2, as suggested in §4; in rows 1, 2, and 3; in rows 1, 2, 3, and 4.

9. Pick out, as indicated in the first column, the combinations of 10 in columns 2-10:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10
7 } 3 }	6	2	6	8	7	9	4	5	1
4 } 9 }	4	8	3	3	8	1	3	2	0
1 } 5 }	2	7	7	9	5	8	2	5	2
5 } 6 }	1	3	5	6	8	7	6	2	8
8 }	7	5	2	5	2	5	7	8	7
2 }	6	4	7	4	9	3	8	8	3
6 }	4	1	6	1	4	7	0	4	9
4 }	8	5	5	7	7	2	1	1	5
9 }	3	4	6	2	4	8	5	6	4
	7	5	4	8	2	6	5	7	6
	3	6	5	1	1	6	9	1	5
	8	3	2	2	3	8	8	9	8
	9	7	8	7	8	5	9	2	8
<u>9</u>	<u>9</u>	<u>7</u>	<u>8</u>	<u>7</u>	<u>8</u>	<u>5</u>	<u>9</u>	<u>2</u>	<u>8</u>

10. Add :

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
8	4	3	5	9	2	6	1	7	5
2	3	7	4	4	7	2	2	6	8
0	8	2	5	0	8	4	8	3	5
7	6	8	3	1	0	5	9	5	1
3	2	3	3	6	3	5	3	4	2
4	7	9	2	3	4	8	6	5	9
6	9	6	1	5	6	2	7	4	2
8	2	1	8	7	5	9	4	5	7
8	8	3	5	6	5	4	5	6	2
7	8	7	5	6	4	7	6	6	5
1	3	4	3	8	1	7	6	6	9
<u>1</u>	<u>3</u>	<u>4</u>	<u>3</u>	<u>8</u>	<u>1</u>	<u>7</u>	<u>6</u>	<u>6</u>	<u>9</u>

11. Read the results in the following. Watch for two-figure combinations and for combinations of 10.

1.	7	2	3	6	2	5	6	4	9	8
	3	7	4	6	9	8	8	7	3	7
	8	8	4	2	3	4	1	9	0	3
	1	1	1	0	1	0	7	3	4	4
	6	4	3	5	4	3	2	8	6	8
	<u>6</u>	<u>4</u>	<u>3</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>8</u>	<u>6</u>	<u>8</u>

2.	5	8	6	2	5	1	7	6	6	2
	6	2	2	8	9	8	3	5	3	3
	2	2	3	9	8	4	7	7	0	9
	9	2	9	4	3	1	5	7	1	4
	2	4	3	9	2	9	9	9	4	8
	<u>2</u>	<u>4</u>	<u>3</u>	<u>9</u>	<u>2</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>4</u>	<u>8</u>

3.	9	9	8	3	4	5	9	6	8	7
	9	8	6	7	0	2	8	8	5	9
	6	7	4	4	4	2	3	0	2	6
	8	2	8	8	8	9	3	7	7	5
	1	7	6	1	2	6	9	3	8	8
	<u>1</u>	<u>7</u>	<u>6</u>	<u>1</u>	<u>2</u>	<u>6</u>	<u>9</u>	<u>3</u>	<u>8</u>	<u>8</u>

4.	1	4	6	7	5	2	3	4	7	8
	0	3	2	5	5	8	7	9	1	6
	2	8	8	4	9	8	5	8	3	5
	0	9	8	2	3	2	9	3	1	7
	8	3	8	9	8	4	3	7	1	4
	<u>8</u>	<u>3</u>	<u>8</u>	<u>9</u>	<u>8</u>	<u>4</u>	<u>3</u>	<u>7</u>	<u>1</u>	<u>4</u>

5.	1	8	4	7	6	2	3	9	5	9
	4	1	8	0	9	2	1	3	9	9
	2	2	6	6	8	9	6	3	6	4
	7	6	8	3	1	6	7	6	8	8
	8	7	6	8	9	5	8	2	7	2
	<u>8</u>	<u>7</u>	<u>6</u>	<u>8</u>	<u>9</u>	<u>5</u>	<u>8</u>	<u>2</u>	<u>7</u>	<u>2</u>

The above results should be read in two minutes or less.

12. Checking is any means of proving the accuracy of the work. If the addition has been made downward, check by adding upward; this is the best means of checking, and usually locates any error.

13. Many accountants use the following method in checking results :

7237		
5641	The sum of the units' column is	21 units
2872	The sum of the tens' column is	24 tens
5436	The sum of the hundreds' column is	28 hundreds
1875	The sum of the thousands' column is	20 thousands
<u>23061</u>		<u>23061</u> (<i>check</i>)

14. The method of checking known as **casting out nines** is sometimes used.

254	2	Add the digits in each addend, dropping the nines as you do so. Drop 4 + 5 (9), and put down the 2. In the second row drop 8 + 1, and put down 0 (no remainder). In the third row, 5 + 8 = 13, drop 9, leaving 4; 4 + 8 = 12 = one 9 and 3 over. The remainder in the fourth row is 7, and in the fifth, 0.
108	0	
885	3	
763	7	
675	0	Adding the several remainders, we get 12 (one 9 and 3 over); dropping the 9's in the result gives a remainder 3. The addition, therefore, is <i>probably</i> correct.
<u>2685</u>	<u>3</u>	

15. In writing or copying numbers, any error caused by transposing the figures of a number is divisible by 9; mistakes may often be located in this way. Thus an error of \$90 might have been caused by writing \$328 instead of \$238 or vice versa; writing 54 instead of 45 gives an error of 9, etc.

WRITTEN WORK

16. Add the following problems by combinations of two figures; remember always to look for sums which equal 10; check results by any method.

1.	2.	3.	4.	5.	6.	7.
538	7622	68498	243967	942268	98254	829328
694	8431	71527	885354	239592	94893	538964
287	6691	42936	476932	873240	67868	932599
436	4424	22679	732369	952823	39675	813836
393	3739	39338	529437	842637	93464	376207
962	7385	78075	952546	749894	74649	933963
452	9461	68627	689475	376846	38473	670244
<u>836</u>	<u>318</u>	<u>24496</u>	<u>396823</u>	<u>367298</u>	<u>72967</u>	<u>592683</u>

8.	9.	10.	11.	12.
2468998	2369857	539863	6892576	6864939682
810773	784972	284006	380919	926784629
3106064	38648	186	6808	16930179
2958	414973	8675	8329727	8202904625
16476	8754	934	986235	623968234
286972	43692	681309	6809763	9813952975
3296498	.57373	372691	4682792	9239392839
968634	865885	458	8257613	289512954
6976	16862	914	398458	6396832689
<u>4163068</u>	<u>6197837</u>	<u>682789</u>	<u>2918975</u>	<u>2139323692</u>

13.	14.	15.	16.
62534785	\$211678.92	4519904	325
42768	4376.53	80022	279002
3426487	52678.21	3428	369001
546891	536.65	319900	32673
63916294	23891.78	538	299751
2354689	996784.46	7723901	6867
552679	8228.77	47892	2871909
64738923	48935.25	72804	4572
634258	243.85	992367	70003
7354629	25364.48	734	247808
300030	114.15	63425	375803
<u>34253645</u>	<u>65.65</u>	<u>6354745</u>	<u>8675391</u>

17.	18.	19.	20.
258014675	334990800	2508146	4466820
39820017	100336472	567	30004
287941	5541763	268749	638914410
47190285	639014725	75271	36819304
832091748	472459016	33728451	591309247
47251635	61538697	163089	1648732
845361924	46573100	2537481	90057338
635127400	361022855	46275541	900036211
714257892	5326178	243885	11922347
364511	56640	273101	4457
38165243	88129934	7142386	553896452
3732450	632871	79573	2004893
992345515	3745106	26374810	7342888
5645	40009786	1118885	375642102
4994494	61195	836	20
437611	1589298	463702	4582901
634459011	2266839	8629815	33561
<u>900026733</u>	<u>63825637</u>	<u>41432678</u>	<u>71280090</u>

TO THE TEACHER. Inability to write or copy numbers accurately is the cause of frequent mistakes. Dictate the above numbers, and then cause the work to be checked for accuracy and legibility. Make frequent use of dictation exercises of this kind.

NOTE. In offices the expression to "foot-up," meaning to "add," is frequently heard ; the sum is called the "footing."

17. Horizontal addition. In invoices and in arrangements of numbers in tabular form, it is sometimes necessary to add numbers horizontally. Care should be taken to combine figures of the same order : units with units, tens with tens, etc. Do not attempt to add horizontally if the numbers are very long ; place a dot over each figure as it is added.

1. Find (*a*) the total native parentage, (*b*) the total foreign or mixed parentage, (*c*) the foreign whites, (*d*) the

total negro population, (e) the total population of each city. Check your work.

1910 CENSUS FOR CITIES OF OHIO

CITY	NATIVE PARENTAGE	FOREIGN OR MIXED PARENTAGE	FOREIGN WHITES	NEGRO POPULATION	TOTAL
Akron	37793	17370	13241	657	
Canton	29470	11798	6848	291	
Cincinnati	154937	132190	56792	19639	
Cleveland	132314	223908	195703	8448	
Columbus	116486	35578	16285	12739	
Dayton	72301	25559	13847	4842	
Hamilton	21866	9371	3309	725	
Lima	23465	4445	1614	978	
Lorain	8455	9112	10929	375	
Newark	19090	3914	1602	1384	
Springfield	30577	8243	3156	4933	
Toledo	75114	59383	32037	1877	
Youngstown	25595	26654	24840	1936	
Zanesville	20885	4145	1602	1384	
Total					

2. Complete the following sales sheet. Add by columns and by rows, and check the work by adding the vertical and horizontal totals and comparing the two amounts.

SUMMARY FOR WEEK ENDING JULY 25

	CARPETS	FURNITURE	STOVES	BOOKS	DISHES	TOTAL
Mon.	\$2436.15	\$8654.72	\$ 237.58	\$ 972.15	\$ 238.17	
Tues.	754.13	2364.58	2364.25	764.73	1324.82	
Wed.	1356.64	1655.18	1293.70	1215.14	2652.73	
Thurs.	592.35	2736.63	75.	836.58	1736.	
Fri.	2317.59	9872.50	4378.56	72.	3642.75	
Sat.	1654.	6375.25	1654.19	2315.04	1568.09	
Total						

3. The number of pupils in school was as follows, as indicated by buildings and by grades. How many were there in each grade? in each building? Check your work.

GRADE	WASHINGTON	LINCOLN	JEFFERSON	FRANKLIN	NORTH	CENTRAL	TOTAL
1	108	62	45	43	94		
2	103	50	42	39	87		
3	95	48	37	39	73		
4	92	43	33	30	65		
5	83	41	29	28	51		
6	72	37	26	25	47		
7	55	31	19	20	40		
8	41	28	17	20	36		
H. S.						834	
Total . .							

SUBTRACTION

18. The most rapid method in subtraction is illustrated as follows: suppose you wish to subtract 3615 from 9521.

9521	Say to yourself: 5 and 6 (put down the 6) are 11, carry 1;
<u>3615</u>	2 (1 + 1 carried) and 0 (put down the 0) are 2; 6 and 9 (put
<u>5906</u>	down the 9) are 15; 4 (3 + 1 carried) and 5 are 9. Put down
	the 5.

In other words you are to find that number which *added* to 3615 will give 9521.

19. Ease and rapidity in subtraction will come with practice. Drill on the following, reading the differences at sight. The eighty results should be read in a minute and a half.

- | | | | | | | | | | | | | | | | | | |
|----|-----------|-----------|------------|------------|------------|------------|-----------|-----------|------------|------------|------------|-----------|-----------|----------|----------|----------|----------|
| 1. | 8 | 4 | 5 | 9 | 6 | 9 | 8 | 9 | 7 | 3 | 8 | 9 | 7 | 4 | 7 | 9 | 8 |
| | <u>2</u> | <u>3</u> | <u>1</u> | <u>5</u> | <u>4</u> | <u>7</u> | <u>4</u> | <u>4</u> | <u>3</u> | <u>3</u> | <u>1</u> | <u>3</u> | <u>6</u> | <u>0</u> | <u>5</u> | <u>3</u> | <u>2</u> |
| | | | | | | | | | | | | | | | | | |
| 2. | 11 | 16 | 13 | 13 | 10 | 11 | 14 | 15 | 17 | 14 | 12 | 13 | 19 | | | | |
| | <u>5</u> | <u>4</u> | <u>7</u> | <u>5</u> | <u>6</u> | <u>7</u> | <u>8</u> | <u>9</u> | <u>8</u> | <u>3</u> | <u>9</u> | <u>8</u> | <u>3</u> | | | | |
| | | | | | | | | | | | | | | | | | |
| 3. | 15 | 19 | 16 | 14 | 18 | 17 | 13 | 18 | 14 | 11 | 19 | 15 | 16 | | | | |
| | <u>4</u> | <u>7</u> | <u>11</u> | <u>8</u> | <u>12</u> | <u>5</u> | <u>11</u> | <u>6</u> | <u>7</u> | <u>4</u> | <u>13</u> | <u>6</u> | <u>13</u> | | | | |
| | | | | | | | | | | | | | | | | | |
| 4. | 21 | 14 | 19 | 23 | 22 | 51 | 20 | 24 | 21 | 20 | 22 | 26 | 21 | | | | |
| | <u>15</u> | <u>4</u> | <u>12</u> | <u>15</u> | <u>17</u> | <u>13</u> | <u>14</u> | <u>19</u> | <u>18</u> | <u>18</u> | <u>13</u> | <u>15</u> | <u>17</u> | | | | |
| | | | | | | | | | | | | | | | | | |
| 5. | 38 | 35 | 41 | 37 | 25 | 42 | 51 | 47 | 81 | 62 | 93 | 46 | 37 | | | | |
| | <u>29</u> | <u>26</u> | <u>38</u> | <u>18</u> | <u>19</u> | <u>26</u> | <u>17</u> | <u>39</u> | <u>79</u> | <u>45</u> | <u>47</u> | <u>32</u> | <u>16</u> | | | | |
| | | | | | | | | | | | | | | | | | |
| 6. | 175 | 205 | 193 | 600 | 750 | 215 | 70 | 195 | 219 | 351 | 514 | | | | | | |
| | <u>68</u> | <u>42</u> | <u>151</u> | <u>480</u> | <u>514</u> | <u>155</u> | <u>57</u> | <u>89</u> | <u>198</u> | <u>248</u> | <u>328</u> | | | | | | |

20. Checking. The best way to check subtraction is to add (upward) the remainder to the subtrahend, checking the result by the figures in the minuend as you do so. Thus in § 18, $6 + 5 = 11$ (check the 1), $0 + 1 + 1 = 2$ (check), $9 + 6 = 15$ (check), $5 + 1 + 3 = 9$.

21. A parenthesis, (), signifies that all quantities within it are to be considered together. A **vinculum** — written over the numbers has the same effect. For example, $17 - (6 + 4)$, or $17 - \overline{6 + 4}$, means that the sum of 6 and 4 is to be subtracted from 17.

22. In making change, add to the amount of the purchase the amount necessary to equal the coin or bill tendered in payment.

1. If a man buys goods amounting to \$2.53, and tenders a twenty-dollar bill, how will the cashier count out the change?

The cashier *picks out*, first 2¢, then 2 dimes, then a quarter, then a \$2 bill, then a \$5 bill, then a \$10 bill (not two 5's), making \$20. The change is *counted out* piece by piece, thus: \$2.53, \$2.54, \$2.55, \$2.65, \$2.75, \$3, \$5, \$10, \$20.

Count out the change in the following :

- | | |
|-------------------|------------------|
| 2. \$5 — \$2.15 | 6. \$2 — \$.72 |
| 3. \$10 — \$1.73 | 7. \$1 — \$.14 |
| 4. \$10 — \$6.13 | 8. \$.75 — \$.52 |
| 5. \$20 — \$11.42 | 9. \$.50 — \$.11 |

COST OF THE ITEMS PURCHASED			AMOUNT TENDERED
10.	\$.25, \$8.25, \$1.25		\$10
11.	\$.14, \$1.52, \$2.11		\$5
12.	\$21.25, \$17.82, \$16.54		\$100

23. To find the arithmetical complement of a number, subtract it from one unit of the next higher order. Thus, the

complement of 8 is 2 ($10 - 8$) ; the complement of 89 is 11 ($100 - 89$).

Arithmetical complements are useful in getting the result in any problem which requires that one number be subtracted from the sum of other numbers.

1. Find the result of 764 minus 121 plus 263.

$\begin{array}{r} 764 \\ - 121 \\ 263 \\ \hline 906 \end{array}$	<p>Arrange as for addition, prefixing a minus sign to the quantity to be subtracted. Add, using the complement of the minus figure in each column; thus, $4 + 9$ (the complement of 1) $+ 3 = 16$; write 6, rejecting one 10, there being one minus number. Continuing, $6 + 8$ (complement) $+ 6 = 20$; write 0, reject 10, carry 1. $8 + 9 + 2 = 19$; write 9.</p>
--	--

(a) When the sum in any column, including the complement, is less than 10, put down the entire sum, and add 1 to the minus figure in the next column before finding its complement.

(b) The complement of 0 is 10.

2. Find the result of \$360.40 minus \$17.08 plus \$114.25.

$\begin{array}{r} \$360.40 \\ - 17.08 \\ 114.25 \\ \hline \$457.57 \end{array}$	<p>$0 + 2$ (complement of 8) $+ 5 = 7$; write 7. Since 10 cannot be subtracted, add 1 to 0, the minus number in the next column, before finding its complement. $4 + 9$ (complement of 0 $+ 1$) $+ 2 = 15$; reject 10, write 5. $0 + 3$ (complement of 7) $+ 4 = 7$. 10 cannot be subtracted. $6 + 8$ (complement of 1 $+ 1$) $+ 1 = 15$; write 5. $3 + 1 = 4$; write 4. There being no minus figure in this column, no 10 is rejected.</p>
---	--

24. Solve the following by complementary addition.

1.	2.	3.	4.	5.
257	\$18.54	\$854.25	328	\$2116.45
196	7.52	- 75.93	- 7	-.58
- 318	54.85	156.55	421	36.71
254	- 10.50	129.62	36	411.95
716	19.87	73.54	880	3246.78
119	5.52	254.76	706	21.16

25. Balancing accounts. The following is taken from a bank ledger. It shows (1) each person's balance at the beginning of the day, (2) the amount drawn out by check, (3) the amount deposited.

1. By horizontal complementary addition determine each person's balance at the close of the day. Check by vertical addition.

DEPOSITOR'S LEDGER

NAMES	BALANCE				CHECKS				DEPOSITS				BALANCE			
Mr. A	2	1	5	7	24		2	8	0		1	5	6	28		
Mr. B		1	6	8			1	2	4	15		2	9			
Mr. C	2	0	0	0				2	0			1	1	0		
Mr. D		1	9	6	25			1	2	25		1	4	8	57	
Mr. E		1	5	4	18			1	0	0		2	4	8	22	
Total																

2. When the debit and credit items of an account are given, this is a convenient method of determining the balance. This is important because of its wide use in bookkeeping.

CASH											
Dr.						Cr.					
	2	5	3	6	24		1	6	8	72	
		6	5	4	25			9	6	8	75
	1	3	6	4	82		2	3	5	6	51
	2	1	4	2	73		1	2	5	4	92
							1	9	4	9	14 Bal.
	6	6	9	8	04		6	6	9	8	04

The debit side of the account is obviously the larger; foot this account and write in the footing, \$6698.04, for *both debit and credit columns*. Add the credit side thus: $2 + 5 + 1 + 2 + (4) = 14$; put down the (4) and carry 1. Continuing, $1 \text{ (carried)} + 7 + 7 + 5 + 9 + (1) = 30$, etc. Check by adding the credit column including the balance.

Solve by the balancing account method :

3. Balance in bank June 1, 1914, \$450. Checks from June 1st to June 30th, \$25.35, \$124.00, \$17.62, \$18.42, \$42.54. What is the balance July 1st?

4. A man bought a farm for \$8260. He built a house on it at a cost of \$1850 and fences cost \$240. What was his gain if he sold it for \$11000?

5. A man's salary is \$1500 per year. If he requires for rent \$300, for personal expenses \$225.38, and for household expenses \$816.27, what amount will he have left at the end of the year?

6. In an election the votes for the candidates A and B in five wards were as follows:

WARD	A	B
1	1261	832
2	793	1116
3	1030	755
4	1349	624
5	998	977
<hr/>		majority

Which candidate has a majority? How large is the majority?

7. Arrange in proper form and find the balance for July 1st. Deposits: Jan. 1, \$300; Feb. 3, \$250; March 2, \$150; June 28, \$250. Checks: Jan. 16, \$125.30; Feb. 7, \$75.20; June 18, \$95.18.

8. A cash register showed the following record of the daily sales of two clerks. Which sold the more goods, and how much?

	A	B
Monday	\$ 43.85	\$ 27.61
Tuesday	78.94	126.13
Wednesday	90.09	81.35
Thursday	148.72	134.25
Friday	110.27	84.16
Saturday	213.50	194.68

Determine the balance in the following :—

9.		10.		11.	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
\$ 236.97	\$ 4657.81	\$ 421.07	\$ 3118.72	\$ 200.	\$ 5000.
2436.18	753.34	1872.56	550.	231.46	428.70
1846.90	624.83	396.54	27.81	61.69	1000.
115.00	524.65	2148.19	642.90	568.89	246.66
198.75	312.40	940.07	168.17	301.01	1587.75
BAL.		BAL.		BAL.	

12.

THE CITY NATIONAL BANK

In Account with *John Fox*

Date		Withdrawn	Deposited	Balance
<i>1914</i>				
<i>Jan</i>	<i>1</i>	<i>Brot ford</i>		<i>25075</i>
	<i>2</i>		<i>37550</i>	
	<i>3</i>	<i>14500</i>		
	<i>5</i>		<i>75000</i>	
	<i>6</i>		<i>22462</i>	
	<i>7</i>	<i>25035</i>		
	<i>10</i>		<i>32454</i>	
	<i>12</i>	<i>7530</i>		
	<i>20</i>	<i>37240</i>		
	<i>27</i>		<i>27242</i>	
	<i>28</i>	<i>42500</i>	<i>34000</i>	
	<i>29</i>		<i>2500</i>	

Write the above from dictation and fill in the balance column. Test the balance for Jan. 30th by the method in exercise 2.

MULTIPLICATION

26. Good work in multiplication depends upon a knowledge of the multiplication tables. Products of numbers up to 15 ought to be recognized at sight.

Thus 9×6 , or 6×9 , should suggest 54 just as readily as $9 + 6$ suggests 15.

27. Drill on the following table until products can be read without hesitation.

As in addition, *think results*.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	2
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	3
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	4
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	5
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	6
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	7
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	8
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	9
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	10
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	11
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	12
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	13
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	14
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	15
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	

28. Multiply 257 by 28.

257
28

2056
514

7196

The product 56 should be suggested without thinking " 8×7 is 56"; write the 6. Next, *think* "forty (8×5) five" (carried); write the 5. Likewise, the next result thought should be 20. Complete the multiplication in this way. In adding the partial products, apply methods for rapid addition.

Multiply :

1.	2.	3.	4.	5.
258	372	156	276	578
<u>37</u>	<u>83</u>	<u>17</u>	<u>46</u>	<u>152</u>
6.	7.	8.	9.	10.
325	768	698	896	1258
<u>247</u>	<u>359</u>	<u>476</u>	<u>754</u>	<u>3267</u>

TO THE TEACHER. Until the correct method is learned, require all work to be done orally.

29. **Checking.** The best method of proving the result correct is to interchange the factors. Thus, in example 9 above, check the product of 896×754 by multiplying 754×896 .

Multiplication may be checked also by casting out the nines. The following example will illustrate the method.

536 5
173 2
1608
3752
536
92728 1

Cast out the nines in the sum of the digits of each factor; find the product of the remainders (5 and 2) and cast out the nine; the remainder (1) should be the same as that found in casting out the nines in the sum of the digits in the result.

Multiply and check :

1. 4672	2. 5736	3. 8245	4. 1213
<u>801</u>	<u>954</u>	<u>3107</u>	<u>1012</u>

RAPID METHODS IN MULTIPLICATION

30. To multiply by 10, 100, 1000, etc. Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.

To multiply by 20, 30, 600, etc. Multiply first by the 2, 3, etc., and then annex the necessary ciphers.

Give results orally :

- | | | |
|---------------------|---------------------|---------------------|
| 1. 24×60 | 4. 350×100 | 7. 724×300 |
| 2. 214×200 | 5. 706×20 | 8. 964×20 |
| 3. 320×30 | 6. 826×60 | 9. 830×70 |

31. Complements (§ 23) may be used in multiplying when the figures of the factors are between 80 and 100.

Multiply 97 by 95.

$\begin{array}{r} 97 \\ 95 \\ \hline 9215 \end{array}$	<p>3 (comp.) Write 15, the product of the complements as illustrated; subtract either complement from the other factor ($95 - 3$ or $97 - 5$) and prefix the remainder to the 15.</p>
--	--

NOTE. If the product of the complements is one figure, prefix 0; if more than two figures, set down the last two figures and carry the other to the next part of the product.

Write the products of the following:

1.	2.	3.	4.	5.	6.	7.	8.	9.
93	98	91	95	86	85	84	98	84
92	91	93	89	96	95	98	97	86

32. To multiply by 11.

Multiply 23724 by 11.

$\begin{array}{r} 23724 \\ 11 \\ \hline 260964 \end{array}$	<p>Write down the right-hand figure (4); write, in order, the sum of the 1st and 2d figures ($4 + 2$), the sum of the 2d and 3d ($2 + 7$), the sum of the 3d and 4th ($7 + 3$), the sum of the 4th and 5th ($3 + 2$) + 1 (carried); write 2, the left-hand figure.</p>
---	--

NOTE. To multiply a number of two figures by 11, write the sum of the two figures between them; thus, $35 \times 11 = 385$.

Multiply :

- | | |
|-------------------|-------------------|
| 1. 43 by 11. | 6. 3824 by 11. |
| 2. 1572645 by 11. | 7. 168549 by 11. |
| 3. 82354 by 11. | 8. 873180 by 11. |
| 4. 9872365 by 11. | 9. 9072 by 11. |
| 5. 1872596 by 11. | 10. 431807 by 11. |

33. To multiply by a number, one part of which is a multiple of another part.

Multiply 265 by 357.

$\begin{array}{r} 265 \\ 357 \\ \hline 1855 \\ 9275 \\ \hline 94605 \end{array}$	<p>The product of 265×7 is 1855; 35 is 5 times 7, hence multiply 1855 by 5 and set down the first figure under the 5 of the 35.</p>
--	---

Multiply 285 by 728.

$\begin{array}{r} 285 \\ 728 \\ \hline 1995 \\ 7980 \\ \hline 207480 \end{array}$	<p>It does not matter which number in the multiplier is first used, if the product is written under the number by which we multiply. Therefore, multiply first by 7, setting down 1995 so that the first figure (5) comes under the multiplier (7); multiply that result (1995) by 4 ($28 = 4 \times 7$), writing 7980 so that the 0 is under the 8 of 28.</p>
---	---

NOTE. In all multiplication work be careful to keep all columns straight, especially when there are a number of partial products.

Multiply : (check)

- | | |
|------------------------|------------------------------------|
| 1. 9512×279 | 7. $342468 \times 24366\backslash$ |
| 2. 8326×327 | 8. 62857×105357 |
| 3. 16547×426 | 9. 165478×9254 |
| 4. 72543×6611 | 10. 625487×72135 |
| 5. 4872×321 | 11. 780855×2408 |
| 6. 5419×355 | 12. 96382×3612 |

EXERCISES

1. 384965 <u>257</u>	2. 84563 <u>355</u>	3. 98376 <u>14503</u>	4. 58764 <u>4267</u>	5. 78456 <u>99011</u>
6. 31528 <u>3024</u>	7. 72349 <u>1608</u>	8. 398107 <u>12372</u>	9. 578655 <u>4657</u>	10. 1394764 <u>10958</u>
11. 2358762 <u>428363</u>	12. 7938767 <u>4305008</u>	13. 72859001 <u>3800</u>	14. 54009768 <u>948358</u>	15. 87210324 <u>7864573</u>

16. A factory employs 2317 men at an average wage of \$2.50 per day. If there are 26 working days in the month of July, what amount will be paid out in wages?

17. The average taken in by a street-car conductor per day is \$53.75. What amount will be taken in by 197 conductors in 31 days?

18. Multiply the sum of 84354 and 17690 by the difference between 53643 and 11319.

19. Following the form in § 27, prepare a multiplication table of numbers from 16 to 25.

20. An automobile manufacturer makes in one year 2678 machines of an average value of \$1275. What is the value of his output?

DIVISION

RAPID METHODS IN DIVISION

NOTE. Special methods depending on fractions will be taken up after that subject has been presented.

34. To divide by 10, 100, 1000, etc.

Beginning at the right-hand figure of the dividend, or at the decimal point, point off as many places to the left as there are ciphers in the divisor.

35. To divide by 30, 400, 5000, and like numbers.

Point off the required number of places, and divide by the left-hand figure of the divisor.

Give results orally :

- | | |
|-----------------------|-------------------------|
| 1. $853427 \div 60$ | 9. $27000 \div 3000$ |
| 2. $25324 \div 300$ | 10. $1425 \div 600$ |
| 3. $742592 \div 1000$ | 11. $3961 \div 70$ |
| 4. $365867 \div 200$ | 12. $7204 \div 90$ |
| 5. $158392 \div 400$ | 13. $170693 \div 30$ |
| 6. $267840 \div 20$ | 14. $554010 \div 800$ |
| 7. $461973 \div 10$ | 15. $913563 \div 100$ |
| 8. $817700 \div 2000$ | 16. $150000 \div 30000$ |

36. To divide by the factors of a number.

Divide 3759 by 21.

$$\begin{array}{r} 3 \overline{)3759} \\ 7 \overline{)1253} \\ \hline 179 \end{array}$$

The factors of 21 are 3 and 7; divide by short division.

Divide 24667 by 105.

$$3 \overline{)24667}$$

$$5 \overline{)8222} + 1$$

$$7 \overline{)1644} + 2 \times 3 =$$

$$234 + 6 \times 5 \times 3 = \frac{90}{97}$$

1

6

97

The factors of 105 are 3, 5, and 7.

Divide by these in turn, setting down the remainder, if any. Multiply each remainder by all previous divisors, and add these products for the final remainder.

NOTE. If the division is exact will there be any remainder at any time during the operation? Why?

37. Checking. The best method of checking division is to multiply the quotient by the divisor and add the remainder; the result should equal the dividend.

38. Division may be checked by **casting out nines**.

Divide 24667 by 105. (See § 36.)

The excess of 9's in the divisor is 6; in the quotient 0; in the remainder 7. 6×0 plus 7 equals 7, the excess in the dividend. The result is probably correct.

EXERCISES

Divide by the factor method:

1. $72348 \div 64$

6. $82492 \div 99$

2. $35678 \div 84$

7. $15436 \div 96$

3. $54792 \div 54$

8. $62743 \div 108$

4. $4594 \div 42$

9. $879654 \div 72$

5. $41687 \div 56$

10. $247859 \div 88$

Divide each of the following by 10; by 100; by 1000; and by 10000:

11. 48000

15. 4900000

19. 20000000

23. 34621

12. 3650000

16. 28490000

20. 21900000

24. 8540

13. 50000

17. 5800000

21. 4758

25. 17543

14. 850000

18. 85000000

22. 73954

26. 75436907

Divide by long division : (check)

27. $749680 \div 768$

32. $379446 \div 5428$

28. $793465 \div 435$

33. $940786 \div 5087$

29. $389576 \div 7820$

34. $987206 \div 7408$

30. $972683 \div 8726$

35. $849752 \div 42785$

31. $698374 \div 5925$

36. $342736 \div 34271$

37. A state's fruit crop of 168,953,967 bushels is valued at \$337,907,934. What is the value per bushel?

38. A grocer's sales for one year were as follows :

January . . .	\$1154.28	July	\$ 990.17
February . . .	877.52	August . . .	1098.51
March	1036.63	September . .	1130.50
April	1124.65	October . . .	1187.98
May	1206.86	November . .	1312
June	1208.90	December . .	1400

What were his average sales per day, allowing 26 days to the month?

MISCELLANEOUS PROBLEMS

As you solve each problem, compare the result with the statement of the problem and decide whether it is sensible.

GROUP 1

1. Add \$294.26, \$17.11, \$300, \$52.25, \$7.64, \$.18, \$3.17, \$1.95, \$219.78, \$1340.05, \$8.01, \$27.17.

2. Brown and Jones's books show the following accounts receivable : \$805.74, \$251.06, \$17.85, \$239, \$84.50, \$371.98, \$726.24, \$73.12, \$304.05, \$420.19, \$35, \$34.21, \$15.10, \$425.50. What amount is due the firm?

3. Find the totals in the following table by horizontal and by vertical addition ; check the work.

ACRES HARVESTED

YEAR	CORN	OATS	WHEAT	POTATOES	TOTALS
1909	3916050	1787496	1827932	212808	
1899	3826013	1115149	3209074	167590	
1889	3189553	1215355	2269585	185393	
1879	3281923	910388	2556134	175487	
Total					

Alternately add and subtract the following :

4. 3758649	5. 98725648	6. \$ 35286.06
<u>2816754</u> (add)	<u>76463580</u>	<u>26471.75</u>
<u>1932935</u> (subtract)	<u>2990031</u>	<u>46209.90</u>
<u>4718264</u> (add)	<u>534200</u>	<u>1000.00</u>
<u>3758927</u> (subtract)	<u>79753546</u>	<u>12994.37</u>
<u>2014896</u> (add)	<u>1010357</u>	<u>1446.46</u>

7. A man purchases a hat for \$3, a pair of gloves for \$1.50, a shirt for \$1.15, four handkerchiefs at 2 for 25¢, 6 collars at 2 for 25¢, and a pair of shoes for \$3.50. If he tenders a twenty-dollar bill in payment, how will the cashier count out the change?

8. Find the result of \$619.20, minus \$24.05, plus \$210.86, plus \$744.08.

9. Find the balance in the following account : debit items, \$2345.28, \$174.29, \$2392.82, \$4412.37 ; credit items, \$861.95, \$340, \$3538.09, \$1746.03.

10. Arrange as in problem 12, page 22, and find the

balance Jan. 1, 1915. *Deposits.* Jan. 1, 1914, \$ 500 ; Mar. 10, \$ 360.25 ; June 19, \$ 195.50. *Checks.* Feb. 18, \$ 54 ; Mar. 23, \$ 27.50 ; June 15, \$ 85.10 ; Nov. 20, \$ 125.

GROUP 2

Multiply :

1.	2.	3.	4.	5.
3547621	500473	97	3740752	9364521
6408	2463	92	642	11
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>

Divide :

6.	7.	8.	9.
218568 ÷ 42	64706 ÷ 434	893458 ÷ 1724	276938 ÷ 2615

10. A planter raises 1368401 pounds of cotton on 2600 acres. What is the average per acre ?

GROUP 3

1. If the remainder is 430, the quotient 1148, and the dividend 876354, what is the divisor ?

2. If 75 acres of land produce 3000 bushels of corn, how many bushels will 140 acres produce ?

3. A grain broker sells 5 cars of corn of 1000 bushels each at 65¢ per bushel. What is the value of the shipment ?

4. B buys chemicals at \$1.85 per 100 pounds. What is the cost of a shipment of 100 barrels weighing 378 pounds each, the weight of an empty barrel being 20 pounds ?

5. A farmer purchased a 125-acre farm for \$11,200 ; he sold 50 acres at \$100 per acre. If he wishes to gain \$1000 on the entire transaction, for how much per acre must he sell the remainder ?

6. What number subtracted 227 times from 826,946 will leave a remainder of 212 ?

7. Arrange as in problem 1, page 20, and determine each person's balance at the close of the day. Check your work.

On March 10, A's balance is \$3224.10, he deposits \$213.47, and checks out \$95.50.

B's balance is \$210, he deposits \$25.75, and checks out \$35.04.

C's balance is \$1000, he deposits \$100, and checks out \$25.

D's balance is \$1305.01, he deposits \$2700, and checks out \$1315.82.

E's balance is \$437.98, he deposits \$105.17, and checks out \$50.

8. A salesman travels 4126 miles at an average cost of $2\frac{1}{2}\text{¢}$ per mile for railroad fare. His trip takes him 41 days, during which time he spends an average of \$4.15 per day for hotels and incidental expenses. What is the cost of the trip?

9. From the following exports of domestic merchandise, find the total number of bushels exported and the total value of the exports.

ARTICLES	QUANTITIES	VALUES
Barley	95437	\$76278
Corn	776984	655805
Oats	46278	24391
Rye	249	254
Wheat	544733	562063
Totals		

10. A manufacturer sells automobiles for \$950 each. He sells 235 machines in March, 341 in April, 462 in May, 508 in June, and 470 in July. If the cost of manufacturing and marketing is \$695.45 for each machine, and the manufacturer replaces guaranteed parts to the total amount of \$864.79, what is his gain on the 5 months' sales?

GROUP 4

1. A campaign committee sent out 125,000 pamphlets. Printing cost \$2.10 per thousand, postage 1¢ each, and addressing and mailing took 33 clerks 5 days at \$2 per day. Find the total cost to the committee.

2. A man pays \$65 an acre for 40 acres of land. He buys at 15¢ each enough nursery stock to plant 80 trees to the acre and pays \$220 for the labor of planting. For how much must he sell the orchard to gain \$1200?

3. A steam shovel removes 57 cubic yards of dirt during each hour of a 10-hour day. Its operating cost is \$13 per day for wages, \$5.25 for fuel and water, and \$6.32 for incidental charges. How much does it cost to remove each cubic yard of dirt?

4. An excursion train of 12 coaches travels 144 miles. Each coach carries 72 passengers who pay \$1.50 each for the trip. How much does the railroad company receive for each mile traveled?

5. A coal dealer bought 475 tons of coal, receiving 2240 pounds for a ton. If he sold it at 2000 pounds per ton, how much did he receive at \$7.50 per ton?

6. A horse consumes 60 bushels of oats in a year, and a cow 40 bushels. A farmer who raises 900 bushels saves out enough to feed 4 horses and 2 cows. At 50¢ per bushel how much does he receive for the remainder?

7. Mr. Frank Carter bought of the Star Grocery:

1 qt. cranberries at 15¢ per qt.

1 basket of grapes at 20¢.

2½ lb. tomatoes at 15¢ per lb.

8½ lb. turkey at 29¢ per lb.

4 lb. butter at 33¢ per lb.

He tendered a twenty-dollar bill in payment. How much change did he receive?

8. Make out, extend, and foot a bill for the following : Chas. Walters bought of the Talmadge Hardware Company, 1 handsaw, \$2.10; 17 pounds of eightpenny nails at 7 ¢ per pound; 25 feet of rubber hose at 18 ¢ per foot; 35 yards of iron sheeting at 25 ¢ per yard; 60 feet of poultry fence at 5½ ¢ a running foot; 2½ gross of ¾ inch screws at 92 ¢ per gross.

9. A dramatic producer spent \$4350 for costumes, \$13,875 for scenery, \$945 for labor, and \$6500 for advertising. His receipts were \$27,500, out of which \$2750 was paid to the author of the play in royalties. How much did he lose ?

10. Determine the balance in the following trial balance of a fire insurance company.

TRIAL BALANCE		
Losses paid	\$16785.90	
Losses not paid	5210.85	
Premiums in hands of agents	7892.54	
Capital		\$ 200000.
Surplus		100000.
Premiums		97500.
Interest		8942.50
Commissions	26847.25	
Taxes	1510.83	
Salaries	7428.10	
General expenses	16582.72	
Investments and loans	290150.69	
Office furniture	2495.10	
Stationery and supplies (inv.)	1828.90	
Accounts receivable	16825.95	
Accounts payable		3180.75
Reserve for losses not paid		5210.85
Organization expenses	1822.03	
Cash		

GROUP 5

1. If 1648 is the divisor and also the quotient in a problem in division, what is the dividend?

2. How many times must 27,685,943 be subtracted from 332,231,316 to leave 0?

3. From the following schedule of property left by a testator, determine the share of each of seven heirs who share equally.

1. Books and library	\$ 2000.	
2. Personal effects	750.	
3. Household furniture	1000.	
4. Account due Home Fur. Co.		\$ 234.50
5. Cash in 1st Nat. Bank	9000.	
6. Notes payable		2340.
7. 750 shares X Y Coal Co.	75000.	
8. Life Ins. Policy	5000.	
9. Loan on same, with int.		1200.
		15.46
10. Improved real estate	85000.	
11. Mortgage on same		12000.
Balance		
	<hr/>	<hr/>
	<hr/>	<hr/>

4. Find the total value of the cotton exports to the following countries: United Kingdom, \$44,580; Canada, \$181,462; Mexico, \$21,205; Central American States, \$236,259; Cuba, \$113,258; Haiti, \$56,206; Brazil, \$10,402; China, \$336,243; Philippine Islands, \$479,655; other countries, \$217,082.

5. A fruit grower receives \$1800 for his apple crop taken from 40 acres of orchard, 60 trees to the acre. His expenses are: for pruning, \$90; for spraying, \$85; for picking and packing, \$185. What is his profit on the crop from each tree?

6. Five children have left to them \$3154 each; one of them dies, and the remaining four divide the entire fortune. How much does each receive?

7. From the following find the total amount of money in circulation in the United States on Sept. 3, 1912. Gold coin, \$611,699,253; gold certificates, \$948,650,439; standard silver dollars, \$71,068,661; silver certificates, \$471,846,931; subsidiary silver, \$146,116,659; treasury notes of 1890, \$2,875,546; U. S. notes, \$338,613,664; national bank notes, \$705,622,027.

8. In the following table of government disbursements, find the total for each month, and the total under each heading:

1912 MONTHS	CIVIL	WAR	NAVY	INDIANS	TOTAL
Jan.	\$ 15702645.70	\$ 11825460.90	\$ 11175174.46	\$ 2132068.60	
Feb.	13562159.76	10476877.55	10750125.72	1353724.31	
Mar.	12824104.11	11371468.21	10689792.99	2028589.16	
Apr.	15906538.27	12098225.20	12830351.60	2018908.63	
May	13399111.59	13245913.70	10068913.22	1423293.35	
June	12098976.24	8463485.21	10053624.53	2051111.91	
July	18698533.93	15134713.55	10923328.68	868803.61	
Aug.	17094698.17	16055472.47	11370147.05	940547.41	
Totals					

9. What is the government's average monthly expenditure for civil purposes? for the war department? for the navy? (Data in example 8.)

10. How much does the civil expenditure exceed the war expenditure for each month? (Data in example 8.)

PROPERTIES OF NUMBERS

39. The **factors** of a quantity are two or more quantities, which, when multiplied together, will produce the given quantity ; *e.g.*, 3 and 5 are the factors of 15.

40. **Factoring** is the process of separating a number or quantity into its factors.

TO THE TEACHER. Give the student much drill on resolving numbers into factors, and on the Tests of Divisibility. Careful work here will save time later in the solution of problems.

41. A **prime number** is one which cannot be exactly divided by any other number except itself and 1.

A **prime factor** is a prime number used as a factor.

Numbers prime to one another are such as have no common divisor other than 1.

42. An **even number** is a number exactly divisible by 2.

43. **Tests of divisibility :**

1. All even numbers are divisible by 2.
2. If the sum of the digits of a number is divisible by 3 or 9, the number is divisible by 3 or 9.
3. If the two right-hand figures of a number are ciphers or are divisible by 4, the number is divisible by 4.
4. If the right-hand figure of a number is 0 or 5, the number is divisible by 5.
5. An even number divisible by 3 is also divisible by 6.
6. If the three right-hand figures of a number are divisible by 8, the number is divisible by 8.
7. When the right-hand figure of a number is 0, the number is divisible by 10.

44. Applying the tests given above, find the prime factors of 6480.

$$\begin{array}{r}
 5 \overline{)6480} \\
 6 \overline{)1296} \\
 6 \overline{)216} \\
 6 \overline{)36} \\
 6
 \end{array}$$

6 is used as a divisor to save time. The prime factors of 6 are 2 and 3. Therefore, $2 \times 3 \times 2 \times 3 \times 2 \times 3 \times 2 \times 3 \times 5$ are the prime factors.

Find the prime factors of :

- | | | | | | |
|-------|--------|---------|-----------|----------|-----------|
| 1. 32 | 4. 135 | 7. 3375 | 10. 74088 | 13. 3540 | 16. 8313 |
| 2. 48 | 5. 144 | 8. 2028 | 11. 15309 | 14. 1016 | 17. 3400 |
| 3. 90 | 6. 676 | 9. 9261 | 12. 11190 | 15. 9664 | 18. 27783 |

45. If the factors are alike, as $3 \times 3 = 9$, 3×3 may be written 3^2 , and read “3 square.” The 2 which is written to the right and a little above the number is called the **exponent**, and indicates the **power** of the number, or the number of times it is to be taken as a factor. $3^3 = 3 \times 3 \times 3$, and is read “3 cube.” 3^4 is read “3 to the fourth power.”

Find the value of :

- | | | |
|----------|-----------|-----------|
| 1. 5^3 | 4. 3^4 | 7. 12^3 |
| 2. 4^2 | 5. 2^5 | 8. 14^2 |
| 3. 7^3 | 6. 21^2 | 9. 8^3 |

10. What does x^2 mean? What does b^3 mean?

11. Find the factors of $x - ax$.

$$\begin{array}{r}
 x \overline{)x - ax} \\
 1 - a
 \end{array}$$

Therefore, the factors of $x - ax$ are x and $1 - a$.

NOTE. No sign has been used between a and x in ax , but multiplication is understood.

When no sign is expressed between two letters or between a number and a letter, multiplication is understood; e.g., 5 a means 5 times a.

Find the factors of

12. $P - PRT$

14. $ax - bx$

16. $5A - 5B$

13. $B - BR$

15. $7 - 7b$

17. $7x - 1x$

18. What does $5(a - b)$ mean?

46. The sign $\sqrt{}$ (called radical sign) indicates that one of the equal factors of a number is to be found, or in other words, that the *square root* is to be taken; *e.g.*, $\sqrt{9}$ means the square root of 9, or 3.

NOTE. For the method of extracting the square root see Appendix.

47. A **common divisor** of two numbers is a number that will exactly divide each of them.

48. The **greatest common divisor** (G. C. D.) of two or more numbers is the largest number which will exactly divide each of them.

49. To find the G. C. D. of two numbers, divide the larger by the smaller, the first divisor by the remainder, and so on until there is no remainder. The last divisor is the G. C. D.

Find the G. C. D. of 576 and 198.

$$198 \overline{)576} (2$$

$$\underline{396}$$

$$180 \overline{)198} (1$$

$$\underline{180}$$

$$18 \overline{)180} (10$$

$$\underline{180}$$

Divide the larger number by the smaller.

The quotient is 2, with a remainder of 180.

Divide this remainder into the first divisor,

198. The result gives a remainder of 18

which goes exactly into 180. The G. C. D.

is the last divisor, 18.

If the numbers are not too large, the process may be shortened as in the following example.

Find the G. C. D. of 66 and 42.

$$42 \overline{)66}$$

$$\underline{24}$$

$$18 \overline{)18}$$

$$\underline{6}$$

Do not indicate all the steps in the division, but merely the remainders. The last divisor, 6, goes into 18 exactly, hence 6 is the G. C. D.

The G. C. D. can often be found by inspection. It must be a factor of the difference between the two numbers; *e.g.*, find the G. C. D. of 24 and 45. The difference between them is 21, whose factors are 3 and 7. Since 7 is not a divisor of both numbers, 3 is the G. C. D.

Find (by inspection whenever possible) the G. C. D. of :

- | | | |
|---------------|-----------------|-------------|
| 1. 57, 95 | 3. 72, 180 | 5. 48, 72 |
| 2. 85, 119 | 4. 63, 45 | 6. 364, 512 |
| 7. 24, 36, 60 | 9. 24, 132, 144 | |
| 8. 144, 111 | 10. 288, 432 | |

11. In a sawmill there are three tree trunks of lengths 32, 48, and 80 ft. What are the longest uniform boards that can be cut from them without waste?

12. A field is 160 rods long and 80 rods wide. Boards of what length under 15 ft. can be used to fence it without waste? (1 rod = $16\frac{1}{2}$ ft.)

50. A **common multiple** of two or more numbers is a number that can be divided by each of them without a remainder; thus, 24 is a common multiple of 2, 3, and 4.

51. The **least common multiple** (L. C. M.) of two or more numbers is the smallest number that can be divided by each of them without a remainder; thus, 20 is the L. C. M. of 2, 4, and 5.

Find the L. C. M. of 14, 49, 77, and 22.

$$\begin{array}{r}
 7 \overline{)14, 49, 77, 22} \\
 11 \overline{) 2, 7, 11, 22} \\
 2 \overline{) 2, 7, 1, 2} \\
 \quad 1, 7, 1, 1
 \end{array}$$

L. C. M. = $7 \times 11 \times 2 \times 7$,
or 1078.

Divide the numbers by any prime factor that will exactly divide two or more of them. Divide the resulting quotient and the undivided numbers in a similar way, continuing until quotients are found that are prime to each other. The product of the several divisors and the last quotients is the L. C. M.

Find the L. C. M. of the following :

- | | | |
|----------------|-------------------|-----------------|
| 1. 9, 18, 45 | 4. 8, 16, 26 | 7. 4, 6, 30, 72 |
| 2. 46, 92, 128 | 5. 36, 45, 72, 90 | 8. 2, 3, 5, 7 |
| 3. 33, 52, 78 | 6. 5, 8, 12, 20 | 9. 81, 120, 117 |

52. Cancellation is useful when the product of several numbers is to be divided by the product of other numbers.

Divide the product of 15×9 by the product of $5 \times 3 \times 2$.

$$\frac{\overset{3}{\cancel{15}} \times 9}{\cancel{5} \times \cancel{3} \times 2} = \frac{9}{2}, \text{ or } 4\frac{1}{2}$$

Divide any number on one side of the line by any number on the other side of the line. When unable to divide further, multiply the numerators together for a new numerator and the denominators for a new denominator.

NOTES. 1. When no uncanceled number on one side of the line will divide an uncanceled number on the other side of the line, divide by any factor common to both.

2. Make use of cancellation whenever possible. Do not multiply or divide numbers until you are sure no cancellation is possible.

Use cancellation in the following problems :

1. Divide $81 \times 25 \times 34 \times 30$ by $21 \times 5 \times 6 \times 17$.
2. Divide $39 \times 91 \times 96$ by $114 \times 95 \times 160$.
3. Divide $64 \times 76 \times 45 \times 68$ by $114 \times 160 \times 30$.
4. A large sheet of paper cuts into 12 small sheets. How many packages of 480 small sheets can be cut from 72,600 large sheets?
5. A plot of ground 320 rods by 400 rods is laid off in lots 50 ft. \times 160 ft. How many such lots can be laid off?
6. 12 cases of books, 140 books to the case, are to be repacked in cases holding only 84 books each. How many cases will be required?
7. If 24 tons of coal cost \$135.75, how much will 60 tons cost at the same rate?

FRACTIONS

53. Fractions are of two kinds, **common** and **decimal**.

54. The difference between common and decimal fractions is that a common fraction may have any number for its denominator, while the denominator of a decimal fraction is always 10, or some power of 10, as 100, 1000, etc.

55. The denominator of a decimal is not written, but is indicated by the number of places to the right of the decimal point; *e.g.*, .125 is read one hundred twenty-five *thousandths* ($\frac{125}{1000}$). 1000 is the denominator because there are three places to the right of the decimal point.

56. (a) Common and decimal fractions are treated together, because speed and accuracy in handling fractions come from the ability to interchange them rapidly.

(b) In the work which follows, the term "fraction" is used to indicate common fractions.

Illustrations:

FRACTIONS		DECIMALS
$\frac{1}{2}$	=	.5
$\frac{2}{5}$	=	.4
$\frac{1}{8}$	=	.125
$\frac{3}{4}$	=	.75

Read the following decimals:

- | | | |
|---------|-------------|--------------|
| 1. .12 | 6. .036 | 11. 1.001 |
| 2. .24 | 7. .202 | 12. 10.1 |
| 3. .225 | 8. .3068 | 13. 548.2375 |
| 4. .31 | 9. 12.35 | 14. 275.125 |
| 5. .3 | 10. 205.001 | 15. 208.08 |

REDUCTION OF FRACTIONS

57. $\frac{2}{3} = \frac{8}{12}$ and $\frac{8}{12} = \frac{2}{3}$.

The *forms* of these fractions have been changed, but not their *values*. $\frac{2}{3}$ has been changed to its equivalent $\frac{8}{12}$ by multiplying both numerator and denominator by 4; $\frac{8}{12}$ has been reduced to its lowest terms $\frac{2}{3}$ by dividing both its numerator and denominator by 4.

NOTE. Decimal fractions cannot be reduced to their lowest terms unless they are changed to the form of common fractions.

58. The factor method.

Reduce $\frac{132}{354}$ to its lowest terms.

$$\begin{array}{l} 2)132 \\ 2)354 \end{array} = \begin{array}{l} 3)66 \\ 3)177 \end{array} = \frac{22}{59}$$

To reduce a fraction to its lowest terms, divide both terms by a common factor; continue to divide by common factors until both terms are prime numbers.

59. The G. C. D. method.

Reduce $\frac{155}{217}$ to its lowest terms.

$$31 \overline{) \frac{155}{217}} = \frac{5}{7}$$

When the common factor cannot be found by inspection, find the G. C. D. by § 49 and divide both terms by it.

60. To reduce a fraction to higher terms.

Change $\frac{5}{6}$ to thirtieths.

$$\begin{array}{l} 30 \div 6 = 5. \\ \frac{5 \times 5}{6 \times 5} = \frac{25}{30} \end{array}$$

The required denominator, 30, is 5 times as large as the denominator 6. Then if the denominator is to be made 5 times as large, the numerator also must be made 5 times as large.

61. To reduce a decimal to its lowest terms.

Reduce .125 to its lowest terms.

$$125 \overline{) \frac{125}{1000}} = \frac{1}{8}$$

Change the decimal to the form of a common fraction and reduce.

Reduce to their lowest terms (by inspection when possible):

- | | | | |
|--------------------|--------------------|---------|------------------------|
| 1. $\frac{25}{40}$ | 4. $\frac{27}{72}$ | 7. .8 | 10. $\frac{81}{123}$ |
| 2. $\frac{8}{80}$ | 5. $\frac{64}{80}$ | 8. .25 | 11. $\frac{144}{1728}$ |
| 3. $\frac{16}{40}$ | 6. $\frac{42}{98}$ | 9. .285 | 12. .5856 |

Reduce :

- | | | |
|-----------------------------|-----------------------------|-------------------------------|
| 13. $\frac{11}{3}$ to 36ths | 16. $\frac{5}{12}$ to 36ths | 19. $\frac{3}{40}$ to 120ths |
| 14. $\frac{3}{9}$ to 27ths | 17. $\frac{2}{7}$ to 21sts | 20. $\frac{3}{16}$ to 128ths |
| 15. $\frac{7}{13}$ to 52ds | 18. $\frac{2}{3}$ to 21sts | 21. $\frac{11}{21}$ to 147ths |

62. A **proper fraction** is a fraction in which the denominator is larger than the numerator, as $\frac{3}{4}$.

An **improper fraction** is a fraction in which the numerator is larger than the denominator, as $\frac{4}{3}$.

A **mixed number** is the sum of a whole number and a fraction, as $2\frac{1}{2}$.

63. To reduce an improper fraction to a whole or a mixed number.

Reduce $\frac{271}{52}$ to a mixed number.

$$\frac{271}{52} = 52 \overline{)271} \quad \text{Divide the numerator by the denominator.}$$

$$\phantom{52 \overline{)271}} \underline{511} $$

Reduce to a whole or a mixed number :

- | | | | |
|---------------------|----------------------|-----------------------|-----------------------|
| 1. $\frac{175}{25}$ | 3. $\frac{115}{35}$ | 5. $\frac{1728}{144}$ | 7. $\frac{1895}{125}$ |
| 2. $\frac{145}{15}$ | 4. $\frac{450}{125}$ | 6. $\frac{1885}{225}$ | 8. $\frac{1020}{126}$ |

64. To reduce a mixed number to an improper fraction.

Reduce $4\frac{2}{3}$ to an improper fraction.

$$\begin{array}{r} 4\frac{2}{3} \\ 3 \\ \hline 12 \\ 2 \\ \hline 14 \end{array} \quad \begin{array}{l} \text{Multiply the whole number by the denominator of the} \\ \text{fraction and add the numerator. Express the result over} \\ \text{the denominator.} \end{array}$$

$$14 \frac{14}{3} = 4\frac{2}{3}$$

Reduce to improper fractions (by inspection when possible) :

1. $5\frac{2}{7}$

3. $5\frac{1}{2}$

5. $16\frac{7}{8}$

7. $24\frac{2}{7}$

2. $8\frac{3}{4}$

4. $17\frac{1}{3}$

6. $75\frac{4}{5}$

8. $1728\frac{1}{2}$

65. Interchange of fractional forms.

(a) Reduce $2\frac{1}{4}\frac{1}{3}$ to an equivalent decimal.

$$\begin{array}{r} .0041 + \\ 243 \overline{) 1.0000} \\ \underline{972} \\ 280 \\ \underline{243} \\ + \end{array}$$

Annex ciphers to the numerator and divide by the denominator. Put the point of the quotient directly above the point of the dividend. Write the first figure of the quotient (4) directly above the last figure of 1000 (the first part of the dividend used). Should there be any vacant places in the quotient between the point and the first number, fill in with ciphers.

(b) Change $.12\frac{1}{2}$ to its equivalent fraction.

$$.12\frac{1}{2} = \frac{12\frac{1}{2} \times 2}{100 \times 2} = \frac{25}{200} = \frac{1}{8}$$

Change the decimal to a common fraction $\frac{12\frac{1}{2}}{100}$. Multiply both numerator and denominator by 2 (the denominator of the fractional part).

Interchange the following (by inspection when possible) :

1. $\frac{1}{2}$

4. $\frac{5}{12}$

7. $\frac{11\frac{3}{4}}{500}$

10. $.37\frac{1}{2}$

13. $.8\frac{2}{5}$

2. $\frac{5}{16}$

5. $\frac{1}{16}$

8. $\frac{4}{1260}$

11. $.11\frac{1}{9}$

14. $.16\frac{2}{3}$

3. $\frac{1}{3}$

6. $\frac{1\frac{2}{5}}{2}$

9. $.6\frac{1}{4}$

12. $.33\frac{1}{3}$

15. $.33\frac{1}{8}$

66. The least common denominator (L. C. D.) of two or more fractions is the L. C. M. of the given denominators.

Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $.33\frac{1}{3}$ to equivalent fractions having the least common denominator.

$$\begin{array}{l} \frac{1}{2} = \frac{6}{12} \\ \frac{2}{3} = \frac{8}{12} \\ \frac{3}{4} = \frac{9}{12} \\ .33\frac{1}{3} = \frac{4}{12} \end{array}$$

The L. C. M. of the given denominators is 12. Reduce each fraction to twelfths.

Reduce to least common denominators :

- | | |
|---|--|
| 1. $\frac{4}{5}, \frac{9}{10}, \frac{1}{2}$ | 5. $.11\frac{1}{9}, \frac{5}{8}, .33\frac{1}{3}, \frac{1}{12}$ |
| 2. $\frac{4}{9}, \frac{8}{27}, \frac{5}{81}$ | 6. $12\frac{2}{5}, 14\frac{2}{3}, 18\frac{9}{15}$ |
| 3. $\frac{11}{48}, .125, .6\frac{1}{4}$ | 7. $.15, .625, .25, .5$ |
| 4. $8\frac{3}{4}, 7\frac{5}{16}, 3\frac{1}{12}$ | 8. $13\frac{2}{3}, 17\frac{1}{2}, 19\frac{3}{4}$ |

ALIQOT PARTS

67. An **aliquot part** of a number or quantity is a number which will divide it without a remainder ; *e.g.*, 2 is an aliquot part of 4.

68. The column under numerator 1 in the following table gives the most useful aliquot parts of 1 ; the rest of the table contains the decimals most frequently found in business and their fractional equivalents. Drill on these until they can be given instantly. The top row gives the numerator ; the column at the left gives the denominator.

TABLE

	1	2	3	4	5	6	7	8	9	10
2	.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
3	$.33\frac{1}{3}$	$.66\frac{2}{3}$	1.00	$1.33\frac{1}{3}$	$1.66\frac{2}{3}$	2.00	$2.33\frac{1}{3}$	$2.66\frac{2}{3}$	3.00	$3.33\frac{1}{3}$
4	.25	—	.75	1.00	1.25	—	1.75	—	2.25	—
5	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	—
6	$.16\frac{2}{3}$	—	—	—	$.83\frac{1}{3}$	1.00	$1.16\frac{2}{3}$	—	—	—
8	$.12\frac{1}{2}$	—	$.37\frac{1}{2}$	—	$.62\frac{1}{2}$	—	$.87\frac{1}{2}$	1.00	$1.12\frac{1}{2}$	—
9	$.11\frac{1}{9}$	$.22\frac{2}{9}$	—	$.44\frac{4}{9}$	$.55\frac{5}{9}$	—	$.77\frac{7}{9}$	$.88\frac{8}{9}$	1.00	$1.11\frac{1}{9}$
10	.10	—	.30	—	—	—	.70	—	.90	1.00
12	$.08\frac{1}{3}$	—	—	—	$.41\frac{2}{3}$	—	$.58\frac{1}{3}$	—	—	—
16	$.06\frac{1}{4}$	—	$.18\frac{3}{4}$	—	$.31\frac{1}{4}$	—	$.43\frac{3}{4}$	—	$.56\frac{1}{4}$	—

69. The aliquot parts of 10, 100, or 1000 can be found from the above table by multiplying the given aliquot parts by 10, 100, or 1000 ; *e.g.*, the first column, expressed as aliquot parts of 10, would be 5, $3\frac{1}{3}$, 2.5, 2, $1\frac{2}{3}$, $1\frac{1}{4}$, $1\frac{1}{9}$, 1, $.83\frac{1}{3}$, $.62\frac{1}{2}$.

EXERCISES

1. Construct tables similar to the table in § 68, showing aliquot parts of 10, 100, and 1000.

What fractional part of 10 is each of the following?

- | | | |
|---------------------|-----------------------|----------------------|
| 2. $2\frac{1}{2}$ | 7. .625 | 12. 1.25 |
| 3. $3\frac{1}{3}$ | 8. $6\frac{2}{3}$ | 13. 2.50 |
| 4. $1\frac{1}{4}$ | 9. $1\frac{2}{3}$ | 14. 5 |
| 5. $.6\frac{1}{4}$ | 10. $1\frac{1}{9}$ | 15. $6.6\frac{2}{3}$ |
| 6. $1.2\frac{1}{2}$ | 11. $1.66\frac{2}{3}$ | 16. 2 |

What fractional part of 100 is each of the following?

- | | | |
|---------------------|----------------------|------------------------|
| 17. $12\frac{1}{2}$ | 22. 12.50 | 27. $16.66\frac{2}{3}$ |
| 18. $33\frac{1}{3}$ | 23. $11\frac{1}{9}$ | 28. 6.25 |
| 19. $16\frac{2}{3}$ | 24. $112\frac{1}{2}$ | 29. $33.33\frac{1}{3}$ |
| 20. $66\frac{2}{3}$ | 25. $116\frac{2}{3}$ | 30. 250 |
| 21. $6\frac{1}{4}$ | 26. 150 | 31. $133\frac{1}{3}$ |

32. What decimals are equivalent to $\frac{1}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{4}{9}$, $\frac{1}{11}$, $\frac{1}{9}$?

33. What fractions are equivalent to .5, $.62\frac{1}{2}$, $.12\frac{1}{2}$, $.44\frac{4}{9}$, $.66\frac{2}{3}$?

34. By inspection change the decimals in the following to fractional equivalents and give the L. C. M. of the denominators: .25, $\frac{5}{12}$, $.33\frac{1}{3}$, $.08\frac{1}{3}$.

35. By inspection change the decimals to fractional equivalents and give the L. C. M. of the denominators: $.37\frac{1}{2}$, $\frac{1}{4}$, $.18\frac{3}{4}$, .5.

36. What fractions are equivalent to $.37\frac{1}{2}$, $.83\frac{1}{3}$, $.62\frac{1}{2}$, $.31\frac{1}{4}$, $.33\frac{1}{3}$, .75, .5, $.22\frac{2}{9}$?

37. What decimals are equivalent to $\frac{1}{6}$, $\frac{2}{9}$, $\frac{1}{8}$, $\frac{5}{6}$, $\frac{7}{6}$, $\frac{1}{4}$, $\frac{9}{4}$, $\frac{4}{3}$, $\frac{3}{2}$?

Solve mentally, and find the total amount in the following.
Use either quantity or price as an aliquot part.

38.

36 yd. @ $12\frac{1}{2}\text{¢}$
60 yd. @ $33\frac{1}{3}\text{¢}$
48 yd. @ $8\frac{1}{3}\text{¢}$
160 yd. @ $6\frac{1}{4}\text{¢}$

39.

81 yd. @ $11\frac{1}{9}\text{¢}$
62 yd. @ $6\frac{1}{4}\text{¢}$
126 yd. @ $66\frac{2}{3}\text{¢}$
144 yd. @ $12\frac{1}{2}\text{¢}$

40.

120 acres @ $\$125\frac{2}{3}$
160 acres @ 150
720 acres @ $112\frac{1}{2}$
36 acres @ $106\frac{1}{4}$

41.

50 yd. @ 48 ¢
72 yd. @ $12\frac{1}{2}\text{¢}$
250 yd. @ 40 ¢
125 yd. @ $\$1.60$
1250 yd. @ 2.40

42.

24 bu. @ $\$1.25$
68 bu. @ 1.50
72 bu. @ 2.50
160 bu. @ 12.50
25 bu. @ .88

43.

75 acres @ $\$120$
150 acres @ 64
 $112\frac{1}{2}$ acres @ 96
 $216\frac{2}{3}$ acres @ 60
640 acres @ $112\frac{1}{2}$

70. Aliquot parts applied to other numbers.

(a) Separate 68 into numbers which are aliquot parts of 60.

$$\begin{aligned} 68 &= 60 + 6 + 2 \\ &= 60 + (\frac{1}{10} \text{ of } 60) + (\frac{1}{3} \text{ of } 6) \end{aligned}$$

(b) Separate 54 into numbers which are aliquot parts of 60.

$$\begin{aligned} 54 &= 60 - 6 \\ &= 60 - (\frac{1}{10} \text{ of } 60) \end{aligned}$$

Separate the following numbers into aliquot parts of 60:

- | | | | | |
|-------|-------|-------|---------|---------|
| 1. 40 | 4. 75 | 7. 45 | 10. 20 | 13. 100 |
| 2. 12 | 5. 80 | 8. 30 | 11. 66 | 14. 10 |
| 3. 55 | 6. 90 | 9. 72 | 12. 120 | 15. 15 |

16. If the interest on a note for 60 days is $\$72$, find the interest for 82 days.

Interest for 60 days = $\$72$

Interest for 20 days = 24 ($20 = \frac{1}{3}$ of 60)

Interest for 2 days = 2.40 ($2 = \frac{1}{10}$ of 20)

Interest for 82 days = $\$98.40$

If the interest on a note for 60 days is \$90, find the interest for :

- | | | |
|---------------|---------------|---------------|
| 17. 50 days. | 21. 48 days. | 25. 80 days. |
| 18. 96 days. | 22. 76 days. | 26. 35 days. |
| 19. 24 days. | 23. 42 days. | 27. 93 days. |
| 20. 120 days. | 24. 135 days. | 28. 180 days. |

29. Find the cost of 80 pounds at 45 ¢ per pound.

$$\begin{array}{ll}
 80 \text{ pounds @ } 50 \text{ ¢} = \$40 & (\frac{1}{2} \text{ of } 80) \\
 80 \text{ pounds @ } 5 \text{ ¢} = 4 & (5 \text{ ¢ is } \frac{1}{10} \text{ of } 50 \text{ ¢}) \\
 80 \text{ pounds @ } 45 \text{ ¢} = \$36 & (\text{By subtraction})
 \end{array}$$

30. Find the cost of 42 yards at $7\frac{1}{2}$ ¢ per yard.

$$\begin{array}{ll}
 42 \text{ yd. @ } 10 \text{ ¢} = \$4.20 & (\frac{1}{10} \text{ of } 42) \\
 42 \text{ yd. @ } 2\frac{1}{2} \text{ ¢} = 1.05 & (2\frac{1}{2} \text{ ¢} = \frac{1}{4} \text{ of } 10 \text{ ¢}) \\
 42 \text{ yd. @ } 7\frac{1}{2} \text{ ¢} = \$3.15 & (\text{By subtraction})
 \end{array}$$

Extend (mentally whenever possible) and foot the following bills:

31.	32.	33.
24 lb. @ $16\frac{2}{3}$ ¢	200 yd. @ 25 ¢	35 bu. @ 60 ¢
50 lb. @ 92 ¢	56 yd. @ 9 ¢	8 bu. @ $62\frac{1}{2}$ ¢
84 lb. @ \$1.75	44 yd. @ 11 ¢	1300 bu. @ 80 ¢
33 lb. @ $33\frac{1}{3}$ ¢	16 yd. @ $7\frac{1}{2}$ ¢	900 bu. @ $66\frac{2}{3}$ ¢
150 lb. @ 40 ¢	15 yd. @ $13\frac{1}{3}$ ¢	1000 bu. @ \$1.03 $\frac{1}{8}$
50 lb. @ 70 ¢	25 yd. @ 55 ¢	480 bu. @ \$1.12 $\frac{1}{2}$
125 lb. @ 12 ¢	116 yd. @ $27\frac{1}{2}$ ¢	480 bu. @ $48\frac{3}{4}$ ¢
18 lb. @ 22 ¢	80 yd. @ $12\frac{1}{2}$ ¢	70 bu. @ \$1.40
563 lb. @ 50 ¢	240 yd. @ 44 ¢	1250 bu. @ 70 ¢
2000 lb. @ $37\frac{1}{2}$ ¢	250 yd. @ 32 ¢	1564 bu. @ 75 ¢
$31\frac{1}{4}$ lb. @ 56 ¢	120 yd. @ 75 ¢	70 bu. @ 50 ¢
96 lb. @ $6\frac{1}{4}$ ¢	250 yd. @ \$1.25	500 bu. @ \$2.50

ADDITION OF FRACTIONS

71. Fractions having the same denominator are added by writing the sum of their numerators over the denominator. Thus, $\frac{2}{7} + \frac{6}{7} = \frac{8}{7}$.

If the denominators are different, reduce the fractions to their least common denominator. Always reduce results to simplest form.

Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{12}$.

$\frac{2}{3} = 8$	By inspection the L.C.D. is 12. Write the 12 as shown in the solution. Find the separate numerators, writing them as shown. Add the numerators, writing the sum as the numerator of 12. "Think" all operations, naming only results.
$\frac{3}{4} = 9$	
$\frac{5}{6} = 10$	
$\frac{7}{12} = 7$	
$\frac{34}{12} = 2\frac{5}{6}$	

72. To add mixed numbers.

(a) Add, $17\frac{1}{4}, 6\frac{1}{6}, 4\frac{5}{8}, 21$

$$30 + \frac{31}{24} = 31\frac{7}{24}$$

Add the whole numbers. Adding the fractions by the above method, we find the sum is $2\frac{1}{4}$. Express the result in its simplest form.

(b) Add, $84.16 = 84.16$
 $164\frac{1}{8} = 164.125$
 $75\frac{2}{3} = 75.666\frac{2}{3}$
 $15\frac{5}{7} = 15.714\frac{2}{7}$

$$339.665\frac{20}{21}$$

Change the common fractions to the decimal form. In changing $\frac{2}{3}$ and $\frac{5}{7}$ to their decimal form, carry out to as many places as there are places in the longest exact decimal. Add the common fractions. Keep the decimal points in a perpendicular line.

NOTE. The fraction $\frac{20}{21}$ in the above is retained only in exact calculations. Ordinarily, if this fraction is more than $\frac{1}{2}$, 1 is added to the nearest figure and the fraction is dropped. The above result would then be 339.666.

Add (by inspection when possible) :

1. $\frac{3}{4}, \frac{2}{3}, \frac{4}{5}$

3. $\frac{5}{12}, \frac{7}{18}, \frac{13}{16}$

5. $\frac{1}{6}, \frac{1}{7}$

2. $\frac{5}{7}, \frac{2}{3}, \frac{2}{5}$

4. $\frac{1}{3}, \frac{1}{4}$

6. $\frac{1}{5}, \frac{1}{3}$

7. .15, 185.6, 106.75

8. 18.195, 17.206, 73.008

9. $15\frac{3}{7}$

$25\frac{1}{3}$

$32\frac{1}{6}$

$58\frac{9}{14}$

$72\frac{5}{6}$

$16\frac{1}{2}$

10. $448\frac{4}{9}$

$562\frac{1}{8}$

$324\frac{1}{16}$

$832.12\frac{1}{2}$

$648\frac{1}{8}$

763

11. $29356\frac{3}{8}$

95465.25

24687.375

$6872\frac{1}{2}$

$58934.53\frac{1}{2}$

$68729\frac{3}{5}$

12. How many yards of silk are there in the following pieces 36^2 , 48^1 , 56^3 , 79 ? (The small numbers written to the right and a little above indicate quarter yards.)

13. How many dozens are there in the following lots, the small numbers indicating twelfths of a dozen: 3^2 , 5^7 , 12^3 , 24^6 , 17^5 , 4^1 , 10 ?

14. How many pounds are there in the following, the small numbers representing sixteenths of a pound: 198^3 , 197^5 , 201^7 , 199^8 , 196^4 ?

SUBTRACTION OF FRACTIONS

73. In subtracting fractions follow the same rules as in adding fractions, only *subtract* the numerators.

(a) Subtract $\frac{1}{4}$ from $\frac{5}{8}$.

$$\begin{array}{r} \frac{5}{8}, 5 \\ \frac{1}{4}, 2 \\ \hline \frac{3}{8} \end{array}$$

Write the common denominator (8) as shown. Change each fraction to the common denominator, and write the numerators (5 and 2) as shown. Subtract the numerators, writing the difference (3) as the numerator of 8.

(b) Subtract 103.27 from $285\frac{1}{8}$.

$$285\frac{1}{8} = 285.125$$

$$\begin{array}{r} 103.27 = 103.27 \\ \hline 181.855 \end{array}$$

The explanation is left to the pupil.

Solve (by inspection when possible):

1. $\frac{5}{6} - \frac{1}{3}$

2. $\frac{3}{4} - \frac{2}{3}$

3. $\frac{9}{10} - \frac{7}{8}$

4. $.668 - .005$

5. $.25 - \frac{1}{8}$

6. $2358.12 - 254\frac{1}{3}$

7. From the sum of $2895.11\frac{1}{9}$ and $3429\frac{1}{3}$ take the sum of 1537.25 and $2405\frac{5}{12}$.

8. Add the difference between $400.37\frac{1}{2}$ and $281\frac{1}{4}$ to the difference between $729.83\frac{1}{3}$ and 425.5 .

9. From the sum of $375\frac{1}{8}$, $728.83\frac{1}{3}$, $.00125$, 42387.135 take $2957\frac{7}{16}$.

10. From 25 take the sum of $9.45\frac{1}{3}$, 3.128 , and 7.25 .

MULTIPLICATION OF FRACTIONS

74. To multiply fractions, multiply the numerators for the numerator of the result, and the denominators for the denominator of the result; *e.g.*, $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$.

Use cancellation whenever possible.

(a) Multiply $\frac{2}{3} \times \frac{9}{14} \times 1\frac{2}{5}$.

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{14}} \times \frac{7}{5} = \frac{3}{5}$$

Follow the rule for cancellation.

(b) Multiply $48 \times .66\frac{2}{3}$.

$$\overset{16}{\cancel{48}} \times \frac{2}{\cancel{3}} = 32$$

$.66\frac{2}{3} = \frac{2}{3}$ by the "aliquot parts" table.

(c) Multiply $28 \times \frac{5}{4}$.

$$\frac{7}{35}$$

$\frac{5}{4}$ is $1\frac{1}{4}$. 1 times 28 is 28; $\frac{1}{4}$ of 28 = 7. Then $\frac{5}{4}$ times 28 equals 35 (28 + 7).

(d) Multiply $28 \times \frac{3}{4}$.

$$\frac{7}{21}$$

$\frac{3}{4}$ is $\frac{1}{4}$ less than 1. $\frac{1}{4}$ of 28 = 7. Subtract 7 from 28.

NOTE. Methods (c) and (d) are very rapid when the numerator is 1 more or 1 less than the denominator. The same principle will be used in division.

(e) Multiply 34.2 by .357.

$$\begin{array}{r} 34.2 \\ .357 \\ \hline 2394 \\ 11970 \\ \hline 12.2094 \end{array}$$

Multiply as in whole numbers. Point off as many places in the product as there are places in both multiplier and multiplicand.

NOTE. To multiply by .01, .001, etc., point off as many places to the left in the multiplicand as there are places in the multiplier.

Multiply (by inspection when possible) :

- | | | |
|--|-------------------------------|--|
| 1. $\frac{89}{144} \times 12$ | 6. $58749 \times .0001$ | 11. $285.4 \times .003$ |
| 2. $18 \times \frac{3}{7}$ | 7. $.564 \times 7.28$ | 12. $99 \times 33\frac{1}{3}$ |
| 3. $21 \times \frac{5}{3}$ | 8. $.45 \times 11\frac{1}{9}$ | 13. 895×9.45 |
| 4. $\frac{4}{11} \times \frac{44}{75}$ | 9. $96 \times .125$ | 14. $\frac{2}{3} \times \frac{6}{7} \times \frac{7}{18}$ |
| 5. $\frac{3}{4} \times \frac{12}{17} \times \frac{34}{39}$ | 10. $36 \times \frac{10}{9}$ | 15. $3216 \times .625$ |

75. To multiply a mixed number by an integer.

Multiply $463\frac{3}{8}$ by 12.

$$\begin{array}{r} 463\frac{3}{8} \\ 12 \\ \hline 8 \overline{)36} \\ 4\frac{1}{2} \\ \hline 926 \\ 463 \\ \hline 5560\frac{1}{2} \end{array}$$

Multiply $\frac{3}{8}$ by 12; multiply 463 by 12. Add the results.

76. To multiply a mixed number by a fraction.

Multiply $248\frac{5}{6}$ by $\frac{6}{7}$.

$$\begin{array}{r} 248\frac{5}{6} \times \frac{6}{7} \\ 7 \overline{)1488} \\ 212\frac{4}{7} \\ \hline \frac{5}{7} \\ \hline 213\frac{2}{7} \end{array}$$

Multiply 248 by $\frac{6}{7}$, by multiplying 248 by 6 and dividing the result by 7.

$$\frac{5}{6} \times \frac{6}{7} = \frac{5}{7}. \quad \text{Add results.}$$

77. To multiply a mixed number by a mixed number.

Multiply $41\frac{3}{4}$ by $28\frac{5}{7}$.

$$\begin{array}{r} 41\frac{3}{4} \\ 28\frac{5}{7} \\ \hline 328 \\ 82\frac{15}{28}, \quad 15 \\ 29\frac{2}{7}, \quad 8 \\ 21 \\ \hline 1198 + 2\frac{3}{8} = 1198\frac{23}{8} \end{array}$$

Multiply 41 by 28.

$$\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}.$$

$$41 \times \frac{5}{7} = \frac{205}{7} \text{ or } 29\frac{2}{7}.$$

$$28 \times \frac{3}{4} = 21.$$

Add results. (See § 72.)

Solve :

1. $342\frac{5}{8} \times \frac{2}{3}$

5. $284\frac{5}{6} \times 273\frac{3}{5}$

2. $\frac{3}{8} \times 281\frac{4}{7}$

6. $466\frac{5}{7} \times \frac{7}{12}$

3. $\frac{5}{11} \times 428\frac{2}{3}$

7. $225.125 \times \frac{8}{9}$

4. $56\frac{3}{8} \times 34\frac{7}{9}$

8. $328.66\frac{2}{3} \times \frac{9}{10}$

MULTIPLICATION OF COMMERCIAL FRACTIONS

78. The fractions most commonly used in business are halves, thirds, quarters, and eighths.

79. Special Cases. To multiply mixed numbers when each fraction is $\frac{1}{2}$.

(a) When the integral parts are the same (squaring).

Multiply $4\frac{1}{2}$ by $4\frac{1}{2}$.

$$\begin{array}{r} 4\frac{1}{2} \\ 4\frac{1}{2} \\ \hline 20\frac{1}{4} \end{array}$$

By partial products, $\frac{1}{2}$ of 4 = 2, and $\frac{1}{2}$ of 4 = 2; combined, they equal 4, the original number. *Multiply the number by itself increased by 1 and add the product of the fractions.*

$$5 \times 4 = 20, \quad 20 + \frac{1}{4} = 20\frac{1}{4}.$$

(b) When the integral parts are different.

Multiply $4\frac{1}{2}$ by $8\frac{1}{2}$.

$$\begin{array}{r} 4\frac{1}{2} \\ 8\frac{1}{2} \\ \hline 38\frac{1}{4} \end{array}$$

Take $\frac{1}{2}$ the sum of the numbers. $\frac{1}{2}$ of $(4 + 8) = 6$. $(8 \times 4) + 6 = 38$.

The product of the fractions = $\frac{1}{4}$. $38 + \frac{1}{4} = 38\frac{1}{4}$.

Application of aliquot parts.

Multiply 650 by 450.

$$\begin{array}{r} 6\frac{1}{2} \\ 4\frac{1}{2} \\ \hline 29\frac{1}{4} = 292500 \end{array}$$

50 is $\frac{1}{2}$ of 100. Then the problem is $6\frac{1}{2}$ hundreds $\times 4\frac{1}{2}$ hundreds. Multiplying the mixed numbers gives $29\frac{1}{4}$ ten thousands ($100 \times 100 = 10000$). $\frac{1}{4}$ of 10000 = 2500. 29 ten thousands (290000) + 2500 = 292500.

Solve orally :

- | | |
|---|---------------------------------|
| 1. $4\frac{1}{2} \times 4\frac{1}{2}$ | 450×450 |
| 2. $3\frac{1}{2} \times 3\frac{1}{2}$ | 35×35 |
| 3. $6\frac{1}{2} \times 8\frac{1}{2}$ | 65 acres at \$85 |
| 4. $5\frac{1}{2} \times 2\frac{1}{2}$ | 55×25 |
| 5. $12\frac{1}{2} \times 12\frac{1}{2}$ | $12\frac{1}{2}$ yards at \$1.25 |
| 6. $9\frac{1}{2} \times 7\frac{1}{2}$ | $.95 \times .75$ |

80. To multiply mixed numbers when the fractions are alike. To the product of the integers add that fractional part of the sum of the integers, and annex the product of the fractions.

(a) When the integral parts are the same (squaring).

Multiply $6\frac{1}{4}$ by $6\frac{1}{4}$.

$$\begin{array}{r} 6\frac{1}{4} \\ 6\frac{1}{4} \\ \hline 39\frac{1}{16} \end{array} \quad \frac{1}{4} \text{ of } (6 + 6) = 3. \quad (6 \times 6) + 3 = 39. \quad \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}. \quad 39 + \frac{1}{16} = 39\frac{1}{16}.$$

Multiply $6\frac{2}{3}$ by $6\frac{2}{3}$.

$$\begin{array}{r} 6\frac{2}{3} \\ 6\frac{2}{3} \\ \hline 44\frac{4}{9} \end{array} \quad \frac{2}{3} \text{ of } (6 + 6) = 8. \quad (6 \times 6) + 8 = 44. \quad \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}. \quad 44 + \frac{4}{9} = 44\frac{4}{9}.$$

(b) When the integral parts are different.

Multiply $4\frac{1}{3}$ by $6\frac{1}{3}$.

$$\begin{array}{r} 4\frac{1}{3} \\ 6\frac{1}{3} \\ \hline 27\frac{4}{9} \end{array} \quad \frac{1}{3} \text{ of } (4 + 6) = 3\frac{1}{3}. \quad (4 \times 6) + 3\frac{1}{3} = 27\frac{1}{3}. \quad \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}. \quad 27\frac{1}{3} + \frac{1}{9} = 27\frac{4}{9}.$$

Application of aliquot parts.

Multiply 625 by 625.

$$\begin{array}{r} 6\frac{1}{4} \\ 6\frac{1}{4} \\ \hline 39\frac{1}{16} = 390625 \end{array}$$

$6\frac{1}{4}$ hundreds $\times 6\frac{1}{4}$ hundreds = $39\frac{1}{16}$ ten thousands.
 $\frac{1}{16}$ of 10000 = 625. 625 annexed to 39 ten thousands
 (390000) gives 390625.

NOTES. 1. When the answer ends in $\frac{1}{16}$, annex 0625 to the integers in the product.

2. When the answer ends in $\frac{9}{16}$, as in $8\frac{3}{4} \times 8\frac{3}{4}$, annex 9 times 0625, which is 5625, to the integers in the product.

Multiply $966\frac{2}{3}$ by $9.66\frac{2}{3}$.

$$\begin{array}{r} 9\frac{2}{3} \\ 9\frac{2}{3} \\ \hline 93\frac{4}{9} = 9344.44\frac{4}{9} \end{array}$$

Express the $\frac{4}{9}$ by as many 4's as there are 6's in the two factors. Point off as in the multiplication of decimals.

NOTE. When the decimal part is substituted for the fractional part, add as many numbers in the answer as were displaced by the aliquot part in both factors. (See $966\frac{2}{3} \times 9.66\frac{2}{3}$.)

81. $3\frac{1}{3}$, $33\frac{1}{3}$, $333\frac{1}{3}$, etc., can be expressed by $\frac{1}{3}$; remember that they are the aliquot parts of 10, 100, 1000, etc.

Solve orally:

- | | |
|--|--|
| 1. $8\frac{1}{4} \times 8\frac{1}{4}$ | 825×825 |
| 2. $10\frac{1}{4} \times 10\frac{1}{4}$ | 1025 bushels at \$10.25 |
| 3. $9\frac{1}{3} \times 9\frac{1}{3}$ | $93\frac{1}{3} \times 933\frac{1}{3}$ |
| 4. $8\frac{3}{4} \times 8\frac{3}{4}$ | 875 doz. at \$8.75 |
| 5. $9\frac{1}{4} \times 9\frac{1}{4}$ | $925 \times .925$ |
| 6. $4\frac{2}{3} \times 4\frac{2}{3}$ | $4.66\frac{2}{3} \times 4.66\frac{2}{3}$ |
| 7. $8\frac{1}{4} \times 4\frac{1}{4}$ | $825 \times .425$ |
| 8. $12\frac{3}{4} \times 12\frac{3}{4}$ | 1275 acres at \$127.5 |
| 9. $3\frac{1}{4} \times 9\frac{1}{4}$ | 325 rugs at \$9.25 |
| 10. $36\frac{2}{3} \times 3\frac{2}{3}$ | 13. 350×350 |
| 11. $3\frac{1}{3} \times 33\frac{1}{3}$ | 14. 286×150 |
| 12. $9\frac{1}{3} \times 9.3\frac{1}{3}$ | 15. 476×175 |

DIVISION OF FRACTIONS

82. The process in the division of fractions differs from that in the multiplication of fractions in that we multiply by the reciprocal of the divisor.

The **reciprocal** of any fraction is the fraction inverted; *e.g.*, the reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$, the reciprocal of 3 ($\frac{3}{1}$) is $\frac{1}{3}$, etc.

83. To divide a fraction by an integer.

(a) Divide $\frac{3}{7}$ by 4.

$$\frac{3}{7} \div 4 = \frac{3}{7} \times \frac{1}{4} = \frac{3}{28}$$

(b) Divide .0042 by 7.

$\begin{array}{r} .0006 \\ 7 \overline{) .0042} \end{array}$ Place the point for the quotient directly above the point in the dividend. Dividing, 7 is contained in 0, 0 times; 7 in 0, 0 times; 7 in 4, 0 times; 7 in 42, 6 times.

NOTE. Observe carefully the position of the decimal point in the above problem.

Divide (by inspection when possible):

1. $\frac{34}{41} \div 17$

9. $.75 \div 5$

2. $\frac{75}{91} \div 15$

10. $.0033 \div 3000$

3. $\frac{24}{35} \div 6$

11. $\frac{204}{351} \div 12$

4. $\frac{11}{15} \div 3$

12. $.7500 \div 15$

5. $\frac{7}{5} \div 5$

13. $\frac{4658}{3257} \div 21$

6. $.12\frac{1}{2} \div 4$ (aliquots)

14. $.88\frac{8}{9} \div 24$

7. $.022 \div 110$

15. $.56\frac{1}{4} \div 18$

8. $.00044 \div 220$

16. $.220122 \div 13$

84. To divide a mixed number by an integer.

When the mixed number is small, change it to an improper fraction and divide as in dividing a fraction by an integer; *e.g.*, $4\frac{1}{2} \div 8 = \frac{9}{2} \div 8 = \frac{9}{16}$.

(a) Divide $6456\frac{2}{3}$ by 7.

$$\begin{array}{r} 922\frac{8}{21} \\ 7 \overline{)6456\frac{2}{3}} \end{array}$$

When the mixed number is much larger than the divisor, divide the integral part of the mixed number as if there were no fraction.

$$2\frac{2}{3} = \frac{8}{3}$$

$$\frac{8}{3} \div 7 = \frac{8}{21}$$

7 is contained in 6456, 922 times with a remainder of 2; then the total remainder will be $2\frac{2}{3}$. Dividing $2\frac{2}{3}$ by 7 gives $\frac{8}{21}$.

(b) Divide 3.40236 by 52.

$$\begin{array}{r} .06543 \\ 52 \overline{)3.40236} \\ \underline{3 \ 12} \\ 282 \\ \underline{260} \\ 223 \\ \underline{208} \\ 156 \\ \underline{156} \end{array}$$

Follow the same method as in (b) § 83.

NOTE. Annex as many ciphers to the dividend as are needed; they have no value; *e.g.*, .5 has the same value as .500. In annexing ciphers be careful not to change the position of the decimal point; *e.g.*, 2 = 2.00.

Divide (by inspection when possible):

1. $4\frac{2}{5} \div 6$

2. $8\frac{5}{6} \div 10$

3. $5\frac{2}{3} \div 7$

4. $686\frac{7}{9} \div 9$

5. $8654\frac{5}{7} \div 7$

6. $956\frac{2}{3} \div 4$

7. $4834\frac{1}{3} \div 11$

8. $478.336 \div 16$

9. $448.71\frac{3}{7} \div 7$

10. $7.672\frac{4}{7} \div 5$

11. $24.66\frac{2}{3} \div 8$

12. $2874.5 \div 100$

13. $587.81\frac{9}{11} \div 11$

14. $34.12\frac{1}{2} \div 8$

15. $5.90\frac{1}{11} \div 11$

16. $48.376 \div 24$

85. To divide any expression by a fraction.(a) Divide $\frac{7}{8}$ by $\frac{3}{4}$.

$$\frac{\frac{7}{\cancel{8}_2} \times \frac{\cancel{4}^2}{3} = \frac{7}{6}}$$

(b) Divide 216 by $\frac{12}{5}$.

$$\frac{18}{\cancel{216}_{12}} \times \frac{5}{\cancel{12}_2} = 90$$

(c) Divide $\frac{3}{4}$ of $\frac{2}{3}$ by $\frac{3}{7}$ of $\frac{4}{9}$.

$$\frac{\frac{3}{4} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{7}{\cancel{3}} \times \frac{\cancel{9}^3}{\cancel{4}_2} = 2\frac{5}{8}}$$

Invert *all* the separate fractions composing the divisor, and cancel.

(d) Divide $7\frac{1}{3}$ by $\frac{11}{15}$.

$$\frac{\frac{22}{\cancel{3}} \times \frac{\cancel{15}^5}{\cancel{11}_2} = 10}$$

Reduce the mixed number to an improper fraction.

(e) Divide .02408 by .172

$$\begin{array}{r} .14 \\ 172 \overline{) 024.08} \\ \underline{172} \\ 688 \\ \underline{688} \\ 0000 \end{array}$$

Move the decimal point in the divisor 3 places to the *right*, to make an integer of it. Therefore, the point in the dividend must be moved 3 places to the *right*. The division is now performed in the same manner as dividing by an integer. (See § 83 b.)

Divide (by inspection when possible):

1. $\frac{3}{17} \div \frac{5}{19}$

9. $440.04 \div 1.1$

2. $\frac{51}{60} \div \frac{17}{20}$

10. $.24 \div 1.11$

3. $25.26 \div \frac{5}{13}$

11. $.0222 \div .0111$

4. $25.6 \div 51.2$

12. $.75 \div .15$

5. $.004 \div .0002$

13. $48 \div \frac{9}{10}$

6. $1\frac{1}{4} \div \frac{2}{3}$

14. $8.5 \div .0017$

7. $3\frac{1}{2} \div .5$

15. $(25\frac{1}{2} \times \frac{5}{17}) \div (\frac{5}{8} \times \frac{4}{9})$

8. $625 \div 2.5$

16. $(\frac{4}{9} \text{ of } \frac{3}{5}) \div (\frac{3}{6} \text{ of } \frac{7}{15})$

86. To divide by a mixed number.

(a) Divide 364 by $2\frac{1}{3}$.

$$\begin{array}{r} 52 \\ 364 \times \frac{3}{7} = 156 \end{array}$$

(b) Divide $6.48\frac{1}{3}$ by $1.3\frac{3}{4}$.

$$\begin{array}{r} 1.3\frac{3}{4} \quad 6.48\frac{1}{3} \\ 12 \quad 12 \\ \hline 16.5 \quad 77.80 \end{array}$$

$$\begin{array}{r} 4.7\frac{5}{33} \\ 16\frac{5}{33} \overline{)77.80} \\ \underline{660} \\ 1180 \\ \underline{1155} \\ 25 \\ 165 = 33 \end{array}$$

The L. C. M. of 4 and 3 is 12. Multiplying both dividend and divisor by 12, gives the new dividend and divisor, 77.80 and 16.5. The division results in a quotient of $4.7\frac{5}{33}$. (See § 83 b.)

(c) Divide $8658.24\frac{11}{148}$ by $.3\frac{2}{3}$.

$$\begin{array}{r} .3\frac{2}{3} \overline{)8658.24\frac{11}{148}} \\ 3 \quad 3 \\ \hline 2361 \quad 3.3\frac{1365}{1628} \\ 11 \overline{)25974\frac{7.2\frac{33}{148}}{148}} \\ 22 \\ \hline 39 \\ 33 \\ \hline 67 \\ 66 \\ \hline 14 \\ 11 \\ \hline 37 \\ 33 \\ \hline 42 \\ 33 \\ \hline 9.33 = \frac{1365}{148}, \quad \frac{1365}{148} \div 11 = \frac{1365}{1628} \end{array}$$

When the fraction in the dividend is too large to handle by example (b) multiply both dividend and divisor by the denominator of the divisor. Divide as in (c) § 85.

Divide:

1. $196\frac{3}{4} \div 6\frac{2}{3}$

2. $4286\frac{1}{3} \div 14\frac{1}{5}$

3. $8694\frac{3}{5} \div 26\frac{1}{2}$

4. $638654\frac{1}{2}\frac{2}{7} \div .39\frac{1}{9}$

5. $64.95\frac{1}{3} \div 16$

6. $83.65 \div 1.7\frac{1}{3}$

7. $689275\frac{1}{2}\frac{6}{9}\frac{1}{1} \div 19\frac{2}{3}$

8. $867 \div 19\frac{2}{3}$

9. $9.37\frac{5}{8} \div .042\frac{2}{3}$

10. $67.45 \div 11\frac{1}{9}$

87. Special methods of division of fractions.(a) Divide 135 by $.12\frac{1}{2}$.

$$\begin{array}{r} 135 \\ 8 \\ \hline 1080 \end{array}$$

$.12\frac{1}{2} = \frac{1}{8}$. Dividing by $\frac{1}{8}$ is the same as multiplying by 8.

(b) Divide 424 by 25.

$$\begin{array}{r} 4.24 \\ 4 \\ \hline 16.96 \end{array}$$

25 is $\frac{1}{4}$ of 100. Pointing off two places divides the number 424 by 100. Dividing by 100 makes the quotient $\frac{1}{4}$ of what it should be, so multiply by 4.

(c) Divide 75 by $\frac{3}{4}$.

$$\begin{array}{r} 75 \\ 25 \\ \hline 100 \end{array}$$

$\frac{3}{4}$ lacks just $\frac{1}{4}$ of itself of being 1. Then *add* $\frac{1}{3}$ of the number to itself.

(d) Divide 80 by $\frac{4}{3}$.

$$\begin{array}{r} 80 \\ 20 \\ \hline 60 \end{array}$$

$\frac{4}{3}$ is just $\frac{1}{3}$ of itself larger than 1. Then *subtract* $\frac{1}{4}$ of the number from itself.

1. $864 \div .16\frac{2}{3}$

2. $264 \div .09\frac{1}{11}$

3. $563 \div .16\frac{2}{3}$

4. $6857 \div 75$

5. $4836 \div 125$

6. $935 \div .88\frac{8}{9}$

7. $36.54 \div 125$

8. $125 \div \frac{5}{6}$

9. $217 \div \frac{7}{6}$

10. $3568 \div .90\frac{1}{11}$

16. $85.93\frac{3}{4} \div \frac{1}{16}$

11. $3568 \div \frac{1}{11}$

12. $3568 \div 90\frac{1}{11}$

13. $50000 \div 250$

14. $9872 \div \frac{4}{5}$

15. $5893 \div .36\frac{4}{11}$

DENOMINATE NUMBERS

88. A **denominate number** is a quantity used in measurements, the value of whose unit has been fixed; *e.g.*, 5 feet and 3 pounds are denominate numbers because the units, feet and pounds, are fixed by law.

89. Some of the subjects to which measurements apply are value, weight, length, area, capacity, time, etc.

NOTE. The tables for the Metric System will be found in the Appendix.

MEASURES OF VALUE

90. United States money.

TABLE

10 mills (m.)	= 1 cent (¢)
10 cents	= 1 dime (d.)
10 dimes	= 1 dollar (\$)
10 dollars	= 1 eagle

NOTE. The mill is not a coin.

The following money is now authorized by the United States Government: —

Coins:

The copper one-cent piece and the nickel five-cent piece.

The silver dime, quarter, half dollar, and dollar.

The gold quarter eagle (\$2.50), half eagle (\$5), eagle (\$10), and double eagle (\$20).

NOTE. Uncoined gold and silver is called bullion. Silver coins less than \$1 are *legal tender* to the amount of \$10; nickel and copper pieces to the amount of 25 cents.

Paper Money :

Silver certificates and gold certificates.

United States notes (greenbacks).

National bank notes.

Treasury notes (not now issued but still in circulation).

91. Canadian money is the legal currency of Canada.

TABLE

10 mills (m.)	= 1 cent (¢)
100 cents	= 1 dollar (\$)

NOTE. Canada also issues subsidiary coins, the silver 5¢ piece, 10¢ piece, 25¢ piece, and 50¢ piece.

92. The unit of English money is the pound sterling ; its value in United States money is \$4.8665.

TABLE

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound or sovereign (£)

93. The unit of French money is the franc ; its value in United States money is \$.193.

TABLE

10 millimes (m.)	= 1 centime (c.)
10 centimes	= 1 decime (dc.)
10 decimes	= 1 franc (fr.)

94. The unit of German money is the mark ; its value in United States money is \$.238.

TABLE

100 pfennigs (pf.)	= 1 mark (M.)
--------------------	---------------

MEASURES OF WEIGHT**95.** There are three kinds of weights used in the United States : Commercial, Troy, and Apothecaries'.**96. Commercial or avoirdupois weight** is used for all commercial weighing, except for weighing precious stones.

TABLE

16 drams (dr.)	= 1 ounce (oz.)
16 ounces	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 cwt. or 2000 lb.	= 1 ton (T.)
112 pounds	= 1 long hundredweight
2240 pounds	= 1 long ton

NOTES. 1. The long ton is used in United States customhouses, and in wholesale transactions in coal and iron.

2. The commercial pound contains 7000 Troy grains.

COMMERCIAL TABLE

Showing pounds per bushel of various products, with exceptions.

COMMODITY	POUNDS		EXCEPTIONS
	U. S. Customhouse	States	
Barley	48	48	Ariz., 45; Ala., Ga., Ky., Pa., 47; Cal., 50.
Beans	60	60	Ariz., 55; N.H., Vt., Me., 62.
Buckwheat	48	52	Cal., 40; Conn., Me., Mass., Mich., Miss., N.Y., Pa., R.I., Vt., 48; Idaho, N. Dak., S. Dak., Okla., Ore., Tex., Wash., 42; Ind., Kan., Minn., N.J., N.C., Ohio, Tenn., Wis., 50.
Clover seed	60	60	N.J., 64.
Corn (in ear) . . .	70	70	Miss., 72; Ohio, Ind., Ky., 68.
Corn (shelled) . . .	56	56	Mass., 50; Cal., 52.
Corn meal	48	50	Ala., Ark., Ga., Fla., Ill., Miss., N.C., S.C., Tenn., 48.
Oats	32	32	Md., 26; N.J., Va., 30; Ida., Ore., 36.
Onions	57	57	Conn., Me., Mass., Minn., N. Dak., S. Dak., Okla., Vt., 52; Fla., Tenn., 56; Ind., 48; Mich., 54; Ohio, 55; Pa., R.I., 50.
Peas	60	60	
Potatoes	60	60	Md., Pa., 56.
Rye	56	56	Colo., 54; Me., 50.
Timothy seed . . .	45	45	Ark., 60; Okla., N. Dak., S. Dak., 42.
Wheat	60	60	

OTHER COMMERCIAL MEASURES

Beef — barrel	200 lb.
Butter — firkin	56 lb.
Fish — quintal	100 lb.
Flour — barrel	196 lb.
Grain — cental	100 lb.
Nails — keg	100 lb.
Pork — barrel	200 lb.
Salt — barrel	280 lb.
Lime — cask	240 lb.

97. Gross weight is the total weight of the goods and the containing package, commonly called container.

98. Net weight is the weight of the goods alone.

99. Tare is the allowance made for the weight of the containing package.

100. Troy weight is used in weighing diamonds, gold, silver, and other precious minerals.

NOTES. 1. The carat used in weighing diamonds is equal to 3.168 grains.

2. The term carat is also used to denote the *fineness* of gold, and means $\frac{1}{24}$ part. Gold 18 K. (carats) fine is $\frac{18}{24}$ pure.

TABLE

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweight = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

TABLE DIAMOND WEIGHT

16 parts = 1 carat grain
4 carat grains = 1 carat (K.)

101. Apothecaries' weight is used by druggists and physicians in compounding and prescribing medicines.

NOTE. Drugs, medicines, and chemicals are bought and sold at wholesale by commercial weight.

TABLE

20 grains (gr. xx)	= 1 scruple (sc. or ʒ)
3 scruples (iij)	= 1 dram (dr. or ʒ)
8 drams (viij)	= 1 ounce (oz. or ʒ)
12 ounces (xij)	= 1 pound (lb or #)

NOTES. 1. In writing quantities in apothecaries' weight, the characters denoting denominations precede the figures, except in pounds. The quantities are usually expressed in Roman characters.

2. Fractions of a pound are generally used instead of ounces.

LONG MEASURE

102. The **statute mile** of 5280 feet is the legal mile in the United States and England.

103. The inch and yard for common use are divided into halves, quarters, eighths, and sixteenths. At the United States customhouses they are divided into tenths, hundredths, etc.

TABLE

12 inches (in. or ")	= 1 foot (ft. or ')
3 feet	= 1 yard (yd.)
5½ yards or 16½ feet	= 1 rod (rd.)
40 rods	= 1 furlong (fur.)
320 rods or 5280 feet	= 1 mile (mi.)

NOTES. 1. Dimensions are written: first, length; then, width; then, height or thickness.

2. The following abbreviations are used: a room 18 ft. long, 14 ft. wide, and 8 ft. 3 in. high may be written: a room 18' × 14' × 8' 3".

SPECIAL LONG MEASURE

1 size	= ⅓ in. Used by shoemakers.
1 hand	= 4 in. Used in measuring the height of a horse.
1 fathom	= 6 ft. Used in measuring depths at sea.
1 knot (geog. mi.)	= 1.152½ mi., or 6086 ft. Used for measuring distances at sea.
3 knots	= 1 league.

104. Surveyors' long measure is used in measuring the dimensions of land, etc.

NOTES. 1. The unit of measure is the Gunter's Chain, which is 4 rods, or 66 feet, long, divided into 100 links.

2. In measuring roads, etc., a tape or chain 100 feet long is used, each foot divided into tenths and hundredths.

TABLE

7.92 inches	= 1 link (l.)
25 links	= 1 rod or pole (rd.)
4 rods or 100 l.	= 1 chain (ch.)
80 chains	= 1 mile (mi.)

SQUARE MEASURE

105. Square measure is used in measuring the areas of surfaces, as land, boards, plastering, etc.

NOTES. 1. The area of a surface is found by multiplying the length by the breadth.

2. Paving, painting, etc., are estimated by the *square* of 100 square feet.

TABLE

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30 $\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

GOVERNMENT LAND MEASURE

106. The public lands are surveyed by selecting a north and south line called a **principal meridian**, and an intersecting east and west line called a **base line**.

107. Range lines are lines running north and south on each side of the principal meridian, at intervals of 6 miles. The strips into which the land is thus divided are called **ranges**.

			5				
			4				
			3				
			2				
			1				
4	3	2	1	1	2	3	4
			1				
			2				
			3				

Principal
Base Line
Meridian

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

A Township divided into Sections

108. Townships are formed by running east and west lines parallel to the base line at intervals of 6 miles.

NOTES. 1. Townships are numbered north and south; ranges, east and west.

2. A township is divided into 36 sections, each 1 mile square. Each section contains 640 acres.

3. A section may be divided into halves or quarters, which are named according to their location in the section; thus, "E. (East) $\frac{1}{2}$ of Sec. 20," "S. W. (South West) $\frac{1}{4}$ of Sec. 20." The halves and quarters may be similarly subdivided; thus, "N. W. $\frac{1}{4}$ of N. W. $\frac{1}{4}$ of Sec. 20."

N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$		E. $\frac{1}{2}$ Section 320 A.
S. $\frac{1}{2}$ of N.W. $\frac{1}{4}$		
S.W. $\frac{1}{4}$ Section 160 A.		

A Section

CUBIC MEASURE

109. Cubic measure is used in measuring the contents or volumes of solids.

TABLE

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
$24\frac{3}{4}$ cubic feet	= 1 perch (P.)
128 cubic feet	= 1 cord (cd.)
1 cubic yard (of earth)	= 1 load

NOTES. 1. A cord of wood is a pile $8' \times 4' \times 4'$.

2. A perch of stone is $16\frac{1}{2}' \times 11\frac{1}{2}' \times 1'$.

3. A cubic foot of water weighs $62\frac{1}{2}$ pounds.

MEASURES OF CAPACITY

110. Liquid measure is used in measuring liquids.

TABLE

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

NOTES. 1. Barrels are of various sizes, but $31\frac{1}{2}$ gallons is a technical barrel.

2. The unit of liquid measure is the gallon of 231 cubic inches.

3. A gallon of water weighs about $8\frac{1}{3}$ pounds.

111. Apothecaries' fluid measure is used by druggists and physicians in compounding and prescribing liquid medicines.

TABLE

60 minims (m.)	= 1 fluid drachm (f 3)
8 fluid drachms	= 1 fluid ounce (f 3)
16 fluid ounces	= 1 pint (O.)
8 pints	= 1 gallon (Cong. = 231 cu. in.)

NOTE. The fluid gallon contains 231 cubic inches.

112. Dry measure is used in measuring grain, fruits, etc.

TABLE

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

NOTES. 1. The unit of measure is the Winchester bushel, which contains 2150.42 cubic inches; it is used in measuring grain, sand, etc.

2. The heaped bushel contains 2747.71 cubic inches; it is used in measuring fruits, vegetables, etc.

3. Fruits and vegetables are often sold by the pound instead of by dry measure.

4. The gallon dry measure contains $268\frac{2}{3}$ cubic inches.

CIRCULAR MEASURE

113. Circular measure is used in measuring angles or arcs of circles.

TABLE

60 seconds (")	= 1 minute (')
60 minutes	= 1 degree (°)
360 degrees	= 1 circle (cir.)

NOTES. 1. The unit of circular measure is the degree, which is $\frac{1}{360}$ of the circumference of a circle.

2. At the equator 1 degree is equal to $69\frac{1}{8}$ statute miles, or 60 geographical miles or knots.

TIME MEASURE

114. The length of the solar year is the exact time required by the earth to make one complete revolution around the sun,—365 days, 5 hours, 48 minutes, 46 seconds (nearly $365\frac{1}{4}$ days).

115. The solar year, divided into 365 days, is called a common year; every fourth year (leap year) 1 day is added to the month of February. A little too much is allowed in this way, so the centennial years not divisible by 400 are excluded. (The year 1900 was not a leap year.)

TABLE

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
100 years	= 1 century (C.)

COMMERCIAL TABLE

30 days	= 1 month (mó.)
12 months	= 1 year (yr.)

116. Standard time is the time adopted by the railroads of the United States and Canada, and by nearly all the people of these countries. The country is divided into four time belts, each extending $7\frac{1}{2}$ degrees east and west from the meridians 75, 90, 105, and 120, west of Greenwich. Since there is just a 15-degree, or 1-hour, difference between each meridian, a difference of 1 hour in time is made between each time belt. The time of the 75th meridian is called Eastern Time; the 90th meridian, Central Time; the 105th meridian, Mountain Time; the 120th meridian, Western or Pacific Time.

MISCELLANEOUS MEASURES

117. Counting.

TABLE

12 units	= 1 dozen (doz.)
20 units	= 1 score
12 dozens	= 1 gross (gro.)
12 gross	= 1 great gross (gt. gro.)

118. Paper.

TABLE

24 sheets	= 1 quire (qr.)
20 quires	= 1 ream (rm.)
2 reams	= 1 bundle (bdl.)
5 bundles	= 1 bale (bl.)

NOTE. Paper is often sold in "reams of 500 sheets."

119. Printers' measure makes use of two units, viz.: point and pica.

TABLE

A point	= $\frac{1}{72}$ of an inch
A pica	= $\frac{1}{6}$ of an inch
The agate	= $5\frac{1}{2}$ points

REDUCTION OF DENOMINATE NUMBERS

120. Descending reduction.

Reduce £ 5 7s. 8d. 2 far. to farthings.

£	s.	d.	far.
5	7	8	2
<hr/>			
20			
107s.			
<hr/>			
12			
1292d.			
<hr/>			
4			
5170 far.			

$$(5 \times 20) + 7 = 107s.$$

$$(107 \times 12) + 8 = 1292d.$$

$$(1292 \times 4) + 2 = 5170 \text{ far.}$$

121. Ascending reduction.

Reduce 59 pt. dry measure, to higher denominations.

$$\begin{array}{r} 2)59 \\ 8)29 \text{ qt.} + 1 \text{ pt.} \\ 3 \text{ pk.} + 5 \text{ qt.} \end{array}$$

Divide 59 by 2 (2 pt. = 1 qt.); divide 29 by 8; result, 3 pk., 5 qt., and 1 pt.

ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

122. (a.) Add:

£	s.	d.	far.	
3	6	5	2	Add (upward) the farthings as follows: 3 + 1 = 4, check because 4 far. make 1d.; continuing, 1 + 2 = 3, write down 3.
	4	6√	1	
	3	4	1√	
6	1	2	3	Add 1d. (carried) + 2 + 4 + 6 = 13 = 1s. 1d., check; continuing, 1 + 5 = 6, write down 6. Perform the rest of the addition in a similar way. Result, £ 9 15s. 6d. 3 far.
9	15	6	3	

(b.) Subtract:

£	s.	d.	far.	
16	8	3	1	Since 3 far. cannot be subtracted from 1 far., borrow 1d., which makes, in all, 5 far.; 5 far. - 3 far. = 2 far. Having borrowed 1d., 2d. is left in the minuend; 2 - 2 = 0. Perform the rest of the subtraction. Result: £ 11 8s. 2 far.
5	0	2	3	
11	8	0	2	

MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS

123. (a). Multiply £ 2 7s. 8d. 2 far. by 12.

£	s.	d.	far.	
2	7	8	2	2 far. \times 12 = 24 far., or 6d. ($12 \times 8d.$) + 6d. =
				102d., or 8s. 6d.; write 6 under the pence.
			12	Perform the rest of the multiplication in a
28	12	6	0	similar way. Result: £28 12s. 6d.

(b). Divide £12 11s. 2d. by 9.

£	s.	d.	far.	
9	12	11	2	Divide £12 by 9 = 1, with a remainder
1	7	10	$3\frac{5}{9}$	of £3. Reduce the £3 to shillings; thus,
				(3×20) + 11 = 71. 71s. \div 9 = 7, rem. 8s.
				(8×12) + 2 = 98, etc. Result: £1 7s. 10d.
				$3\frac{5}{9}$ far.

EXERCISES

1. Change £4 5s. to United States money.
2. How many farthings are there in 200 marks?
3. Divide £18 17s. 11d. by 15.
4. What is the cost of 7 bu. of cherries at 12¢ a quart?
5. Find the cost of 2 lb. 10 pwt. of ore at 8¢ per grain.
6. Multiply 24 bu. 3 pk. 5 qt. by 16.
7. Reduce 3 T. 5 cwt. 15 lb. 10 oz. to drams.
8. What is the value of a diamond weighing $\frac{5}{16}$ of a carat at \$125 per carat?
9. Reduce 38 lb. to grains, apothecaries' weight.
10. A farm is 24 ch. 15 l. long, and 32 ch. 14 l. wide. How many rods of fence will be required to inclose it?
11. Add: 1 mi. 85 rd. 5 yd., 3 mi. 17 rd. 4 yd. 2 ft. 9 in., 4 mi. 17 rd. 1 ft. 10 in.
12. Read: S. $\frac{1}{2}$ of S.E. $\frac{1}{4}$ Sec. 33. *Pat ex*

13. How many acres are there in a field 200 rd. long and 27 rd. wide?

14. How many barrels of water can be contained in a tank $8' 3'' \times 3' 9'' \times 8\frac{1}{2}'$?

15. From a farm containing 45 A. 72 sq. rd., a lot $35' \times 165'$ was sold. How much land was left?

16. A man wishing to travel abroad changed \$100 to English money, \$100 to French money, and \$100 to German money. How much money of each kind did he receive?

17. A man bought a farm located as follows: N. $\frac{1}{2}$ of N.E. $\frac{1}{4}$, Sec. 4, T. 15, North R. 5 E. How many rods of fence are required to inclose it?

18. Find the value in United States money of the following:

BOOKS	£	s.	d.
Lodge of Edinburgh		10	6
Defoe's Robinson Crusoe		4	6
Robert Burns		6	6
Philosophical Works of Bacon		10	6
Henley on Burns		2	0
Early Printing		3	6
Freemasonry		6	6
Postage		10	0
Total			

19. Extend and foot the following bill:

$\frac{1}{2}$ oz. iodoform	@	70		
1 lb. abs. cotton		30		
1 pt. cresol comp.		50		
$\frac{1}{4}$ lb. bismuth subnitrate		60		
$\frac{1}{4}$ lb. bismuth subgallate		60		
Total				

20. A man bought a portrait in Paris, paying 12114 francs 8 centimes for it. What was its value in United States money?

21. How many barrels of flour can be made from 2000 bu. of wheat, if 1 bu. will make 42 lb. of flour?

22. What is the value of an 18-carat gold case, weighing 50 pwt., at 80¢ per pennyweight of pure gold?

23. Together four men own $228\frac{3}{8}$ bu. of potatoes. If A owns 75 bu. 3 pk., B owns 52 bu. $3\frac{1}{2}$ pk., and C owns $17\frac{1}{3}$ bu., how much does D own?

24. Reduce £ 27 17s. 8d. 2 far. to farthings.

25. How much are £ 25 worth in United States money?

26. How many dollars are there in 30 marks?

27. How many pounds sterling are there in 500 marks?

28. A man bought 5 pairs of gloves at 5 francs per pair, and a hat for 15 francs. He tendered in payment £ 2. How many francs did he receive in change?

29. How many pounds will 1000 silver dollars weigh, 1 dollar weighing $412\frac{1}{2}$ gr.?

(30. A coal dealer bought 1000 T. of coal at \$6 per long ton, and retailed it at \$8 per short ton. What was his gain?

31. A merchant bought goods amounting to £ 6000 8s. After selling part of the goods for £ 5000 6s. 6d., he found he had remaining one third of the original purchase. Did he gain or lose on the part sold, and how much?

32. A pupil gets the following grades in his school work for the first term: English, 83; arithmetic, 89; history, 92; science, 84. Find the average grade of his work for the term.

33. Find the average yearly production of oats in the United States from 1900 to 1910, the figures given being millions of bushels: 809, 736, 987, 784, 894, 953, 964, 754, 807, 1007, 1126.

34. A grocer bought 50 qt. of cranberries at 9¢ per quart, dry measure. He sold them at 11¢ per quart, liquid measure. How much more did he gain than he would have gained had he sold them by dry measure?

35. An automobile consumes, on the average, $1\frac{1}{2}$ gal. of gasoline per day. If gasoline costs 18¢ per gallon, find the cost of the gasoline consumed from Feb. 5th to Nov. 26th, if the machine is run 6 days out of every 7.

36. A farmer sold 3 loads of potatoes containing respectively 54 bu. 17 lb., 42 bu. 41 lb., and 60 bu. 28 lb. What did he receive at 80¢ per bushel?

37. A farmer sold 5 loads of wheat, the weights in pounds of the various loads being as follows:

WEIGHT OF WAGON LOADED	WEIGHT OF WAGON
7525	3870
7308	3728
7000	3709
6968	4000
7628	3910

How many pounds did he sell?

DENOMINATE NUMBERS BY ALIQUOT PARTS

124. Grain is generally priced by the bushel, but sold by the pound.

1. Find the value of 24000 lb. of wheat at 95¢ per bushel.

\$240, value at 60¢ per bushel
 120, value at 30¢ per bushel
 20, value at 5¢ per bushel
 \$380, value at 95¢ per bushel

There are 60 lb. in a bushel of wheat; then 60¢ per bushel would be 1¢ per pound. Point off two places in the number of pounds to find the value at 60¢ per bushel = \$240. $\frac{1}{2}$ of \$240 would give the value at $\frac{1}{2}$ of 60¢, or 30¢ per bushel; $\frac{1}{5}$ of \$120 would give the value at $\frac{1}{5}$ of 30¢, or 5¢ per bushel.

2. What is the value of 24000 lbs. of coal (retail) at \$8 per ton ?

24.000
4
\$96.

Point off 3 places to show the number of thousands of pounds. Multiply by $\frac{1}{2}$ the price, for \$8 per ton is \$4 per 1000 pounds.

3. Extend and foot the following bill of feed :

No. of Lbs.	Article	Price	Value
84654	Hay	\$14 per ton	
12570	Bran	\$18 per ton	
546570	Corn (in ear)	40¢ per bushel	
685470	Straw	\$6 per ton	
40020	Chop	\$25 per ton	
68400	Oats	30¢ per bushel	
Total			

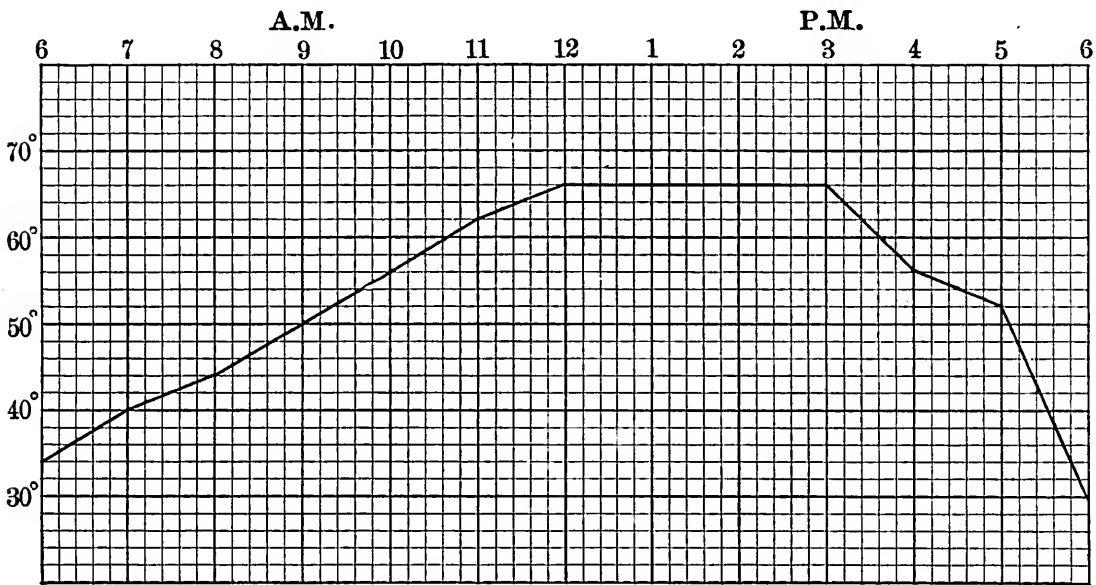
4. Find the value of the following shipment :

Produce	Weight in Lbs.	Price per Bu.	Value
Wheat	76350	75¢	
Wheat	74000	70¢	
Clover seed	36540	\$3.00	
Potatoes	75640	90¢	
Onions	32400	76¢	
Beans	48600	85¢	
Corn (shelled)	78400	49¢	
Barley	38000	30¢	
Timothy seed	48200	\$2.00	
Rye	60000	56¢	
Total			

GRAPHS

125. A **graph** is a diagram showing fluctuations, such as variations in temperature, rise and fall of prices, etc.

The temperature chart shows variations in temperature from 6 A.M. to 6 P.M. The hour is indicated by the row



TEMPERATURE CHART

of figures along the top ; the temperature is indicated by the vertical column at the left.

Illustrate graphically :

1. The varying price of wheat for the years 1900 to 1911, the prices for the respective years being as follows : 1900, $61\frac{1}{2}\phi$; 1901, $63\frac{1}{8}\phi$; 1902, $67\frac{1}{2}\phi$; 1903, $70\frac{1}{4}\phi$; 1904, $81\frac{1}{4}\phi$; 1905, $77\frac{7}{8}\phi$; 1906, $69\frac{1}{8}\phi$; 1907, 71ϕ ; 1908, $84\frac{1}{2}\phi$; 1909, $99\frac{1}{4}\phi$; 1910, $89\frac{1}{2}\phi$; 1911, $83\frac{1}{4}\phi$.

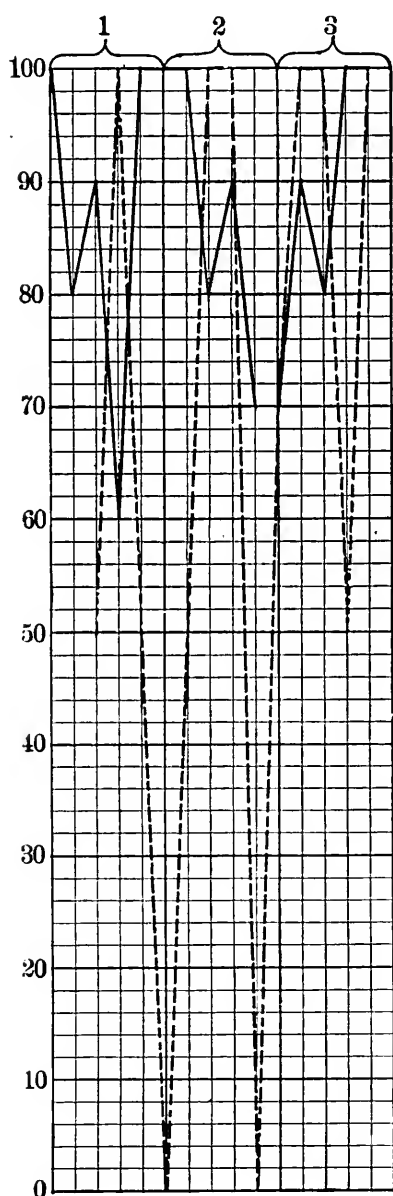
2. The production of corn in the United States for the years 1900 to 1910, the figures given indicating millions of bushels : 1900, 2105 ; 1901, 1522 ; 1902, 2523 ; 1903, 2244 ;

1904, 2467 ; 1905, 2707 ; 1906, 2927 ; 1907, 2592 ; 1908, 2668 ; 1909, 2772 ; 1910, 3125.

TO THE PUPIL. On a sheet of graph paper keep a daily record, after your work has been checked in class, of problems solved correctly.

Problems missed should be worked after they have been explained in class. Then, by means of a *dotted* line, indicate the *percentage* of problems so corrected. This line should not show much variation.

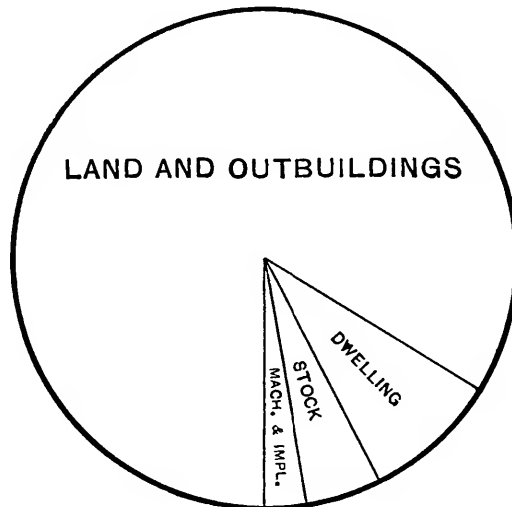
3. The following graph illustrates a record kept for three weeks :



WEEK	DAY	WORKED	MISSSED	MADE UP	% MADE UP
1	Mon.	10	0		
	Tues.	8	2		
	Wed.	9	1	1	50
	Thurs.	6	4	2	100
	Fri.	10	0	2	50
2	Mon.	7	0	0	0
	Tues.	6	0	1	50
	Wed.	8	2	1	100
	Thurs.	9	1	2	100
	Fri.	7	3	0	0
3	Mon.	7	3	3	75
	Tues.	9	1	4	100
	Wed.	8	2	1	100
	Thurs.	8	0	1	50
	Fri.	7	0	1	100

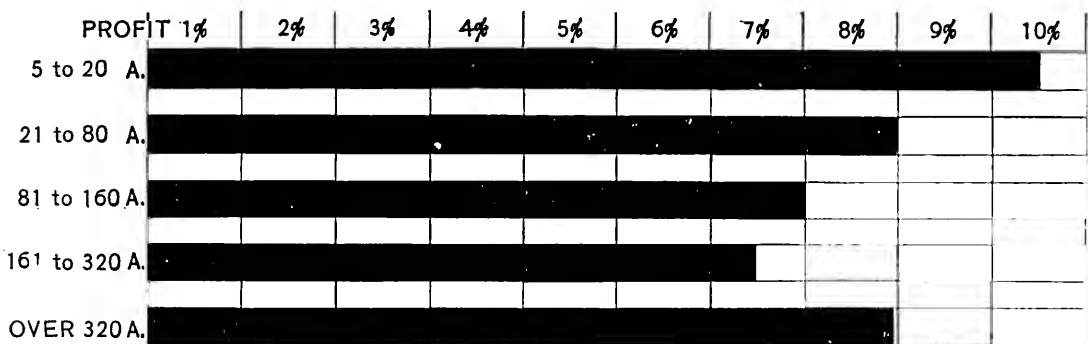
EXPLANATION. On Monday, first week, no problems were missed, so the solid line starts at 100 %. On Tuesday, 80 % of the assignment was correct. On Wednesday, one problem of those missed on Tuesday was made up (50 %). On Thursday, the remaining problem missed on Tuesday and the one missed on Wednesday were worked (100 %), etc.

4. The following illustration shows in a graphical way the percentages of investment on an 80-acre farm :



Land and Outbuildings	83 $\frac{1}{3}$ % of the total investment
Dwelling	8 $\frac{1}{3}$ %
Stock	5 $\frac{5}{9}$ %
Machinery and Implements	2 $\frac{7}{9}$ %

5. The following is a graphical representation of the profits of Oregon farmers on farms of different size.



6. A farmer who keeps a careful estimate of the cost and returns on all his crops finds that wheat makes him 7% profit, corn 6 $\frac{1}{2}$ %, oats 5 %, and apples 10 %. Illustrate his profits, graphically, as in example 5.

7. Draw a graph similar to the one found in example 4, which will show approximately the following family expenditures.

	Heat and				
	Rent	Household	Clothes	Light	Miscellaneous
% of income spent	20	25	15	5	30

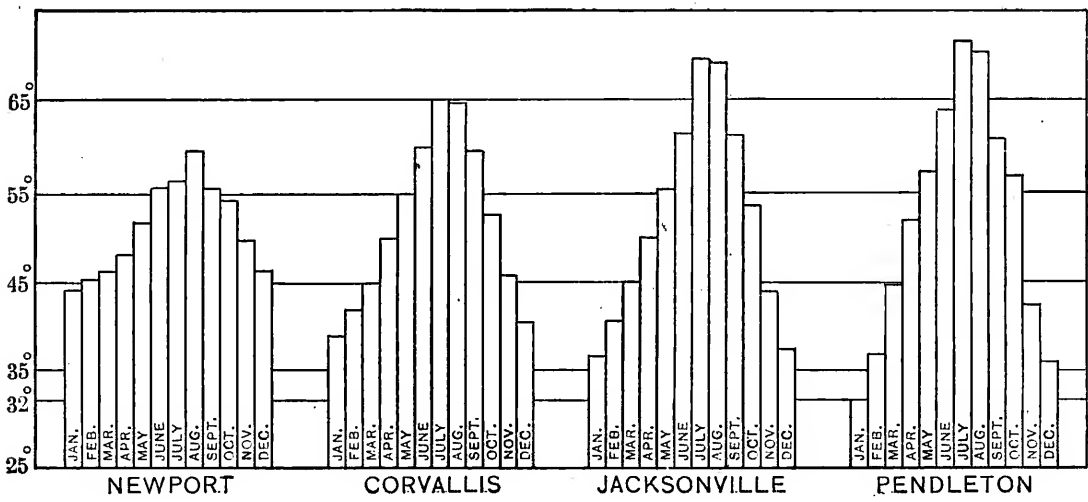
Is the balance which is saved, represented on the graph ?

8. On a farm of 320 acres the following percentages of investment are made:

Land and Outbuildings	75 % of total investment
Dwelling	3 %
Stock	19 %
Machinery and Implements	3 %

Represent, approximately, the above percentages.

9. The following is a graphical representation of the “ Mean Monthly Temperatures ” for certain towns in Oregon as shown by the Oregon Agricultural College.



1. What towns have the same March temperature ?
2. Which town has the coldest January temperature ?
3. What is the coldest temperature recorded ?

10. Represent in a way similar to that used in example 9 the "Mean Monthly Rainfall" as shown in the following statistics:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Town A	10 in.	$8\frac{1}{2}$ in.	7 in.	5 in.	4 in.	2 in.	$\frac{1}{2}$ in.	$\frac{1}{2}$ in.	$2\frac{1}{2}$ in.	$5\frac{1}{2}$ in.	8 in.	7 in.
Town B	6 in.	$5\frac{1}{2}$ in.	4 in.	$2\frac{1}{2}$ in.	2 in.	1 in.	$\frac{1}{4}$ in.	$\frac{1}{4}$ in.	$1\frac{1}{2}$ in.	3 in.	6 in.	6 in.
Town C	1 in.	1 in.	1 in.	$\frac{1}{2}$ in.	1 in.	$\frac{1}{4}$ in.	$\frac{1}{8}$ in.	$\frac{1}{8}$ in.	1 in.	1 in.	$\frac{1}{2}$ in.	$\frac{1}{2}$ in.

MISCELLANEOUS PROBLEMS

GROUP 1

- Find the prime factors of 7892, of 5280, and of 754.
- Find the factors of $c + cr$, of $6x + 6y$, and of $pr + p$.
- Find the G. C. D. of 126, 140, and 168.
- A farmer has 4 farms containing respectively 160 A., 240 A., 280 A., and 300 A. If he divides the farms into equal parcels containing the largest number of acres possible, how many acres will there be in each parcel?
- Find the L. C. M. of 16, 64, 80, 144.

Use cancellation in the following :

- Divide $78 \times 96 \times 54 \times 160$ by $26 \times 16 \times 56 \times 72$.
- Divide $12\frac{1}{2} \times 66\frac{2}{3} \times 100$ by $11\frac{1}{9} \times 62\frac{1}{2} \times 16\frac{2}{3}$.
- A man bought a piece of ground 380 rd. long and 140 rd. wide, paying \$125 an acre. How much did it cost him?
- A man buys a plot of ground 100 rd. long, lying between two streets 335 ft. apart. How much does it cost him at \$500 an acre?
- How many cords of wood are there in a pile 70 ft. long, 10 ft. high, and 6 ft. wide?

GROUP 2

- 1. Solve mentally, $6\frac{1}{2} \times 6\frac{1}{2}$; 550×550 ; $.85 \times .65$; $\frac{2}{3} \times 8\frac{2}{3}$.
- 2. Add $94\frac{1}{8}$, $152\frac{1}{6}$, $736\frac{1}{4}$, $55\frac{3}{4}$, $92\frac{1}{3}$, and 14.5.
- 3. Multiply $362\frac{4}{7}$ by $563\frac{2}{3}$.
- 4. Divide 87.478 by 3.84.
- 5. Divide $94358.13\frac{1}{3}$ by $.18\frac{2}{3}$.
- 6. Divide 15 by $.12\frac{1}{2}$; by $.16\frac{2}{3}$; by $.11\frac{1}{9}$; and by $.62\frac{1}{2}$.
- 7. Extend and foot the following bill :

203 yd. muslin @ $11\frac{1}{8}\phi$	22	58
197 yd. cotton goods @ $5\frac{1}{4}\phi$	10	34
201 yd. madras @ 17ϕ	34	17
108 yd. denim @ $13\frac{1}{2}\phi$		08
216 yd. silk @ 92ϕ	198	72

- 8. Extend and foot the following bill :

72 $\frac{3}{4}$ lb. cheese @ $17\frac{1}{4}\phi$		
113 $\frac{1}{2}$ lb. butter @ $25\frac{1}{2}\phi$		
203 $\frac{1}{4}$ lb. live geese @ $12\frac{1}{2}\phi$		
63 lb. ducks @ $13\frac{1}{2}\phi$		

- 9. I bought 348 eggs for \$4.35, and sold them at 24¢ per dozen. How much did I gain?
- 10. How much will $4\frac{1}{2}$ bu. of apples cost at 25¢ per half peck?

GROUP 3

- 1. What aliquot part of $112\frac{1}{2}$ is $12\frac{1}{2}$? What part of $\frac{9}{8}$ is $\frac{1}{8}$?
- 2. What fractional part of a number is the number plus $\frac{3}{8}$ of itself. What fractional part of 200 is 275?
- 3. What is the cost of five 60-lb. tubs of butter at $33\frac{1}{3}\phi$ per pound?

4. Extend and foot the following bill :

7½ lb. galv. 3d nails @ 6½¢		
3¾ lb. sash cord @ 35¢		
24 8-lb. weights @ \$1.40 per hundredweight		
16 sash locks @ 75¢ dozen		
1½ doz. W. S. bolts @ 20¢ dozen		
17 pr. 3½ × 3½ butts @ 12½¢		
10 door locks @ 33⅓¢		
5 oz. S. C. irons @ 6¼¢ ounce		
25 lb. putty @ 4½¢		
12 24" × 30" glass @ 52½¢		
7 12" × 28" glass @ 16⅔¢		
3½ lb. brads @ 8½¢		

5. After making the proper form, extend and foot the following bill :

On July 10, 1913, Howe Bros. sold to M. S. Scott, 3½ dozen ½-qt. pans 6¾" × 2¼", at 22¢ per dozen ; 15 1½-qt. pans 7¾" × 3", at 33⅓¢ per dozen ; 5 wash boilers 22⅝" × 12¼", at \$13.50 per dozen ; 9 8-qt. galv. pails, at \$1.35 per dozen ; 6 12-qt. pails at \$1.75 per dozen.

6. After making the proper form, extend and foot the following bill :

On Oct. 21, 1913, the Richardson Rug Co. of Chicago, sold to McAllister and Co., Vincennes, Ind., 24 Axminster rugs 27" × 54", at \$1.50 each ; 18 Axminster rugs 30" × 60", at \$3.25 each ; 9 Wilton velvet rugs 36" × 72", at \$2.75 each ; 16 Tapestry Brussels rugs 27" × 54", at \$.75 each ; 24 Body Brussels rugs 6' × 9', at \$12.50 each ; 6 Body Brussels rugs 9' × 12', at \$20 each ; 44' of hall runner 27" wide, at 33⅓¢ per ft.

7. The interest on \$250 for 60 da. is \$2.50. Find the interest for 45 da. ; for 75 da. ; for 130 da. ; for 190 da.

Make the extensions mentally, and find the total amount in each :

8.

- 12 yd. at $12\frac{1}{4}\text{¢}$
- 96 yd. at $6\frac{1}{4}\text{¢}$
- 45 yd. at 10¢
- 25 yd. at 19¢
- 26 yd. at 15¢

9.

- 1 gal. at \$1.90
- $\frac{1}{2}$ gal. at 98¢
- $2\frac{1}{4}$ gal. at \$1.90
- $3\frac{1}{2}$ gal. at \$2.50
- $12\frac{1}{2}$ gal. at \$5.50

10. Extend and foot the following bill :

2½ doz. 9'' plates @ \$1.85 per ½ dozen		
6⅔ doz. 10'' plates @ \$2.20 per dozen		
10 10½'' plates @ \$2.95 per ½ dozen		
1¼ doz. 8½'' plates @ \$1.35 dozen		
½ doz. 7½'' plates @ \$1.35 dozen		

GROUP 4

- 1. A man works overtime 1 hr. and 15 min. each day for 3 da., and 1 hr. and 40 min. each day for 5 da. At 36¢ per hour for overtime, how much extra has he earned in the 8 da.?
- 2. A race horse trots a mile in 2:04. How many yards does it travel in $10\frac{2}{5}$ sec.?
- 3. A railroad gains in elevation 1232 ft. in a distance of 6 mi., 3520 ft. What elevation is gained per mile?
- 4. An automobile circles a half-mile track in 44 sec. How many feet does it travel per second? What is its speed per hour?
- 5. A train travels a mile in 52 sec. What is its speed per hour?
- 6. A dealer bought 22 gal. of milk at 13¢ per gallon, and sold it 5¢ per glass of $\frac{1}{2}$ pt. What is his gain?
- 7. I owe £13 8s. 7d. in England, 1000 francs in France,

and 324 marks in Germany. What is my total indebtedness in United States money?

8. What is the gain on 2500 yd. of English woollens bought at 3s. 6d. per yard, and sold at \$1.40 per yard?

9. What is the gain in U.S. money on a French automobile costing 8000 francs, if sold in England for £420?

10. A druggist bought 200 lb. (commercial weight) of chemicals at 31¢ per pound. What is his gain if he sells at the rate of 42¢ per pound (apothecaries' weight)?

GROUP 5

ORAL WORK

1. Add $\frac{1}{6}$, $\frac{2}{3}$, $\frac{3}{5}$.

2. Multiply $6\frac{1}{2}$ by $6\frac{1}{2}$; 650 by 650

Multiply $5\frac{1}{2}$ by $3\frac{1}{2}$; 5.5 by 350

Multiply $8\frac{1}{2}$ by $7\frac{1}{2}$; .85 by .75

Multiply $6\frac{1}{4}$ by $6\frac{1}{4}$; 625 by 625

Multiply $9\frac{1}{3}$ by $6\frac{1}{3}$; $6.33\frac{1}{3}$ by $9.3\frac{1}{3}$

Solve :

3. $.06\frac{1}{4} \div 4$

4. $25 \div .12\frac{1}{2}$

5. $75 \times .75$

6. $60 \times 3\frac{1}{3}$

$.125 \div 5$

$32 \div .25$

$45 \times .11\frac{1}{9}$

$15 \div .3\frac{1}{3}$

$.033 \div 110$

$48 \div .06\frac{1}{4}$

$30 \times .33\frac{1}{3}$

$12 \times .62\frac{1}{2}$

$.3458 \div 1000$

$75 \div .06\frac{2}{3}$

$2400 \times .08\frac{1}{3}$

$.12\frac{1}{2} \div .66\frac{2}{3}$

$42.68 \div 200$

$.72 \div .08\frac{1}{3}$

$6400 \times .16\frac{2}{3}$

$.6\frac{2}{3} \times 5$

7. What will be the cost of 25 bath robes at \$2.50 each?

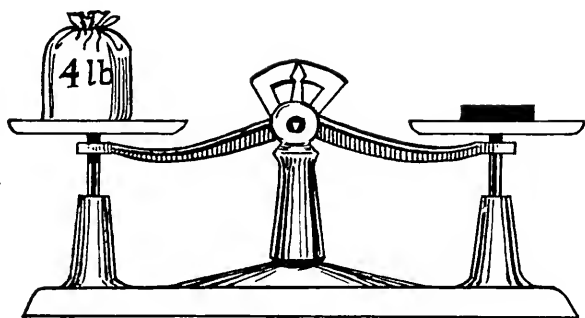
8. What will be the cost of 15 squares of gravel roofing at \$1.50 per square?

9. A retailer bought \$40 worth of cotton goods at $6\frac{1}{4}$ ¢ per yard, and sold the lot at $12\frac{1}{2}$ ¢ per yard. How much did he make?

10. I bought 300 articles at $33\frac{1}{3}$ ¢ each, and sold them at 50¢ each. How much did I gain?

THE EQUATION

126. If a four-pound weight in one pan of the scales exactly balances a quantity of sugar in the other pan, we may say that in respect to weight, the four-pound weight is



equal to the sugar. A statement that one quantity is equal to another quantity is called an **equation**.

The *sign of equality* ($=$) is read “equals.”

127. The equation is used for abbreviating statements: Thus, the question, “If a 50-acre farm is worth \$5000, what is 1 acre worth?” can be abbreviated to read: “If $50\text{ A.} = \$5000$, what does A. equal?”

Further, the statement that, “If 50 acres are worth \$5000, 1 acre is worth 1-50th of \$5000, or \$100,” can be abbreviated to read:

If $50\text{ A.} = \$5000$,
A. = \$100, value per acre.
(By dividing by 50.)

128. If $50\text{ A.} = \$5000$, what will $100\text{ A.} = ?$

If $50\text{ A.} = \$5000$,
 $100\text{ A.} = \$10000$.
(By multiplying by 2.)

129. Further illustrations might be given to indicate that any operation may be performed on both sides of an equa-

tion without destroying its value. From this fact we derive the following important rules:

(a) **Dividing** both sides of an equation by the same number does not alter the value of the equation.

(b) **Multiplying** both sides of an equation by the same number does not alter the value of the equation.

(c) **Adding** the same number to both sides of an equation does not alter the value of the equation.

(d) **Subtracting** the same number from both sides of an equation does not alter the value of the equation.

(e) **Squaring** both sides of an equation does not alter the value of the equation.

(f) **Taking the square root** of both sides of an equation does not alter the value of the equation.

130. If, in the equation $50 A. = \$5000$, the value of $A.$ ($\$100$) is put in place of $A.$, we get the following statement:

$$50 \times \$100 = \$5000.$$

This process is called **substitution**. It is used in checking the work, and in the application of formulas. (See § 147, page 94.)

131. Solution of problems by the equation. In the equation $50 A. = \$5000$, we found that $A. = \$100$, or we say that we have solved the equation for the value of $A.$

1. Solve for x in the equation $4x = 20$.

Dividing by 4, $x = 5$.

If the division cannot be performed, indicate it thus:

Solve for x in the equation $bx = 20$ (bx means b times x).

Dividing by b , $x = 20 \div b$, or $\frac{20}{b}$.

2. Solve for x in the equation $\frac{1}{2}x = 20$.

Multiplying by 2, $x = 40$.

3. Solve for x in the equation $2x + 5 = x + 15$.

Subtracting 5, $2x = x + 10$.

Subtracting x , $x = 10$.

4. Solve for x in the equation $2x - 4 = 16$.

Adding 4, $2x = 20$.

Dividing by 2, $x = 10$.

Substituting 10 for x , $(2 \times 10) - 4 = 16$ (*check*).

NOTE. Add or subtract until the left-hand side of the equation has only the term containing the unknown.

5. Solve for x in the equation $x^2 = 16$.

Extracting square root, $x = 4$.

6. Solve for x in the equation $\sqrt{x} = 3$.

Squaring, $x = 9$.

Solve and check :

7. $3x = x + 10$

8. $7x + 10 = 5x + 14$

9. $\frac{1}{3}x = 3$

10. $9x - 10 = 2x + 11$

11. $\frac{5}{2}x = 15$

12. $\frac{2}{3}b = 4$

13. If $PR = 20$, find R .

14. If $ab = c$, find b .

15. If $PR = I$, find R .

16. If $PRT = I$, find T .

17. If $PRT = I$, find R .

18. If $PRT = I$, find P .

19. If $P + PRT = A$, find R .

SUGGESTION.

$$P + PRT = A.$$

Subtracting P ,

$$PRT = A - P.$$

Dividing by PT ,

$$R = \frac{A - P}{PT}.$$

20. If $x + ax = y$, find x .

SUGGESTION.

$$x + ax = y.$$

Factoring,

$$x(1 + a) = y.$$

Dividing by $1 + a$,

$$x = \frac{y}{1 + a}.$$

21. If $P + PRT = A$, find P .

22. If $B + BRT = A$, find B .

23. If $B + BRT = A$, find R .

24. If $B + BRT = A$, find T .

25. If $B = 3$ and $A = 12$, find h in $Bh = A$.

SUGGESTION.

$$\text{If } Bh = A,$$

Substituting,

$$3h = 12.$$

Then,

$$h = 4.$$

26. If $h = 4$ and $A = 16$, find B in $\frac{1}{2} Bh = A$.

27. If $B = 3$ and $h = 4$, find H in $H^2 = B^2 + h^2$.

28. If $h = 3$, $k = 12$, $K = 16$, and $A = 40$, find P in $Ph + k + K = A$.

29. If $K = 15$ and $V = 45$, find h in $\frac{1}{3} hK = V$.

30. If π (called pi) = 3.1416 and $A = 28.2744$, find r in $\pi r^2 = A$.

If $a = 3$, $b = 4$, and $c = 5$, find x in the following:

31. $x + a = c$

37. $a^2 + x = b^2 + c$

32. $2x + b = x + 2a$

38. $x^2 = 4a + b$

33. $x - a = 3b$

39. $b = x^2$

34. $c + 3x = 5a - b$

40. $ax = bc$

35. $2a + 4x = 6b$

41. $bx = ac + 9$

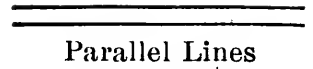
36. $a + b + c = 3x$

42. $2x - 10 = abc$

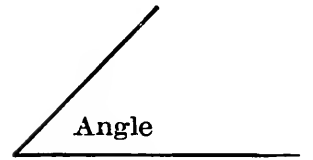
MENSURATION

132. A **line** has one dimension, length.

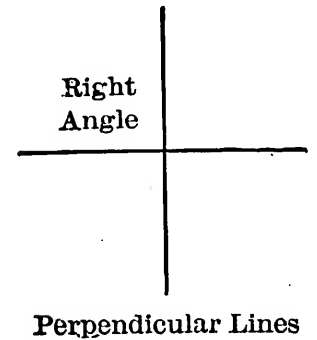
NOTE. **Parallel lines** have the same direction.



133. An **angle** is the difference in direction of two lines that meet.



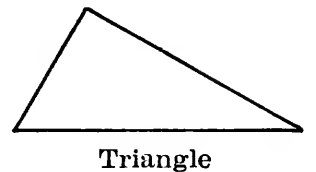
134. If two lines cross or meet, one being perpendicular to the other, they form **right angles**.



PLANE FIGURES

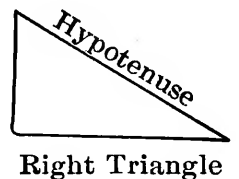
135. A **polygon** is a plane surface bounded by straight lines. Polygons derive their names from the number of their sides, as triangle, quadrilateral, etc.

136. A **triangle** is a plane surface having three sides and three angles.



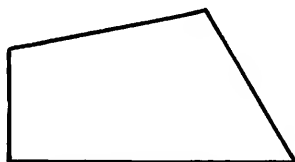
137. A **right triangle** is a triangle having a right angle.

NOTE. The side opposite the right angle is called the **hypotenuse**.

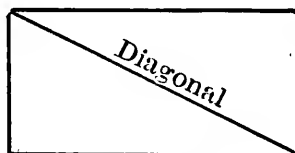


138. A **quadrilateral** is a plane surface having four sides and four angles.

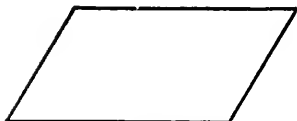
- NOTES. 1. When the sides are parallel, it is called a **parallelogram**.
2. If the parallelogram has right angles, it is called a **rectangle**.
3. If a rectangle has equal sides, it is called a **square**.



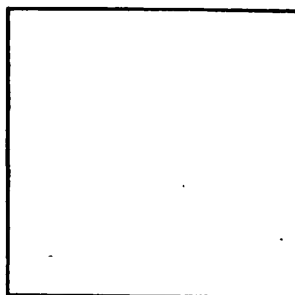
Quadrilateral



Rectangle



Parallelogram



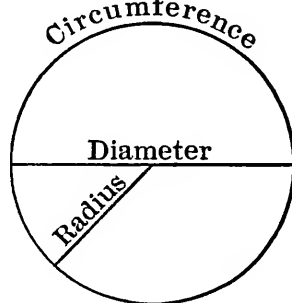
Square

139. The **base** of a plane figure is the side upon which it is supposed to stand.

140. The **altitude** is the perpendicular distance from the base to the vertex (highest point of the figure).

141. A **circle** is a plane surface bounded by a curved line (**circumference**), every point of the circumference being equally distant from the center.

142. The **diameter** of a circle is a straight line drawn through the center, terminating in the circumference.



143. The **radius** of a circle is the distance from the center to the circumference, or half the diameter.

144. The **perimeter** of a plane surface is the distance around it.

145. The **area** of a plane figure is the number of square units within its boundary lines.

146. Abbreviations used :

h	= altitude.
B	= base.
a, b, c	= the three sides of a triangle.
P	= perimeter.
S	= $\frac{1}{2}(a + b + c)$.
H	= hypotenuse.
C	= circumference.
D	= diameter.
r	= radius.
π (pi)	= 3.1416.

147. Formulas for plane figures :**1. PARALLELOGRAM,**

$$B \times h = \text{area.}$$

(The base multiplied by the altitude equals the area.)

TO THE TEACHER. Require the pupil to interpret all the formulas.

2. TRIANGLE,

(a) Base and altitude being given,

$$B \times \frac{1}{2} h = \text{area.}$$

(b) The three sides (a, b, c) being given,

$$\sqrt{S(S-a)(S-b)(S-c)} = \text{area.}$$

Ex. Find the area of a triangle if the three sides are 15 ft., 17 ft., and 18 ft.

$$S = \frac{1}{2}(15 + 17 + 18) \text{ or } 25.$$

$$\sqrt{25 \times 10 \times 8 \times 7} = 118.3^+ \text{ sq. ft.}$$

(c) Two sides of a given right triangle to find the third.
(Either leg may be considered the base.)

$$(1) \quad H^2 = B^2 + h^2$$

$$H = \sqrt{B^2 + h^2} \text{ (For taking square root, see Appendix.)}$$

$$(2) \quad B^2 = H^2 - h^2 \text{ (Subtracting } h^2 \text{ from both sides in the first equation.)}$$

$$B = \sqrt{H^2 - h^2}$$

$$(3) \quad h^2 = H^2 - B^2 \text{ (Why?)}$$

$$h = \sqrt{H^2 - B^2}$$

3. CIRCLE,

$$D \times \pi = \text{circumference.}$$

$$C \div \pi = \text{diameter.}$$

$$\frac{1}{4}(D \times C) = \text{area.}$$

$$r^2 \times \pi = \text{area.}$$

$$D^2 \times \frac{1}{4} \pi \left(\frac{1}{4} \pi = .7854 \right) = \text{area.}$$

EXERCISES

Find the area and perimeter of:

1. A rectangle, base 72 rd., width 30 rd.
2. A square, base 25 rd., width 25 rd.
3. A rectangle, base 30 rd., width 42 rd.
4. A circle, radius 3 ft.
5. A circle, diameter 4 ft.
6. A circle, circumference 24 ft.
7. Find the area of a triangle whose sides are 15', 16', and 17'.
8. Find the base of a right triangle whose hypotenuse is 10 ft. and one side 6 ft.
9. What is the hypotenuse of a right triangle whose other two sides are 9 ft. and 12 ft.?

10. Find the area of a right triangle, if its base and perpendicular are 7 ft. and 8 ft. respectively.

11. In a right triangle the hypotenuse is 25 ft., the base 13 ft. What is its area?

12. A rectangular field is 70 rd. long and has an area of 40 A. What is its width?

13. How many square feet in the floor of a room 40 ft. by 20 ft.?

14. A yard is 75 ft. by 165 ft. What will it cost to fence it at $12\frac{1}{2}$ ¢ per foot?

15. If a walk 3 ft. wide is inside the fence of a rectangular garden, how much area does the walk cover, the garden being 30 ft. by 40 ft.?

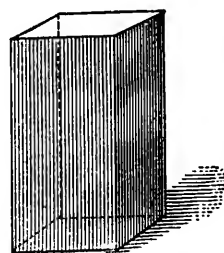
16. A square field containing 40 A. has a road 30' wide running completely around it outside the fence. Find the area of the road in acres and square rods.

SOLIDS

148. A **solid** has three dimensions, — length, breadth, and thickness.

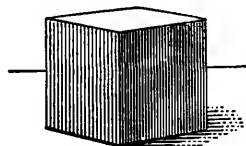
149. A **prism** is a solid whose ends or bases are any equal, similar, and parallel plane figures, and whose lateral faces are parallelograms.

NOTE. Prisms are named from the number of sides forming their bases, — triangular prisms, quadrilateral prisms, etc.



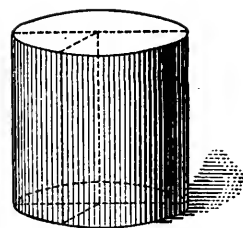
Quadrilateral
Prism

150. A **cube** is a prism having all three dimensions equal.



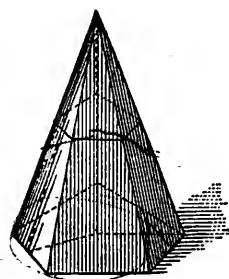
Cube

151. A **circular cylinder** is a solid having equal, parallel, circular bases, and its lateral surface a uniform curve.



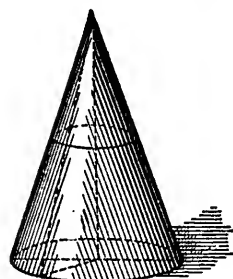
Circular Cylinder

152. A **pyramid** is a solid having a polygon for a base and triangles for sides.



Pyramid

153. A **circular cone** is a solid having a circular base, and lateral surface tapering to a point.



Circular Cone

154. The **altitude** of a solid is the perpendicular distance from its base to the highest point.

155. The **slant height** of a pyramid is the altitude of its triangular lateral surface.

156. The **slant height** of a cone is the shortest distance from the circumference of its base to its vertex.

157. The **volume** of a body is the number of cubic units it contains.

158. Additional abbreviations:

K = area of lower base.

k = area of upper base.

st ht. = slant height.

159. Formulas for solid figures:**1. PRISM,**

$$(a) K \times h = \text{volume.}$$

$$(b) P \times h = \text{area of lateral surface.}$$

$$(c) (P \times h) + k + K = \text{total area.}$$

2. CYLINDER,

$$(a) K \times h = \text{volume.}$$

$$(b) P \times h = \text{area of lateral surface.}$$

$$(c) (P \times h) + k + K = \text{total area.}$$

3. PYRAMID,

$$(a) K \times \frac{1}{3} h = \text{volume.}$$

$$(b) P \times \frac{1}{2} \text{ st. ht.} = \text{area of lateral surface.}$$

$$(c) (P \times \frac{1}{2} \text{ st. ht.}) + K = \text{total area.}$$

4. CONE,

$$(a) K \times \frac{1}{3} h = \text{volume.}$$

$$(b) P \times \frac{1}{2} \text{ st. ht.} = \text{area of lateral surface.}$$

$$(c) (P \times \frac{1}{2} \text{ st. ht.}) + K = \text{total area.}$$

NOTES. 1. The prism and cylinder have like formulas.

2. The pyramid and cone have like formulas.

3. In formulas 1 (c) and 2 (c), $K = k$.

EXERCISES

Find the volume, lateral area, and total area of :

1. A rectangular prism, sides of base 3 ft. and 4 ft., altitude 9 ft.

2. A triangular prism, sides of base 4 ft., altitude 9 ft.

3. A cylinder, diameter of base 5 in., altitude 10 in.

4. A cylinder, radius of base 2 in., altitude 8 in.

5. A pyramid, base a square 6 ft. on a side, altitude 4 ft., slant height 5 ft.
6. A cone, diameter of base 4 in., altitude 7 in.
7. A triangular prism, sides of base 4 in., 5 in., and 7 in., altitude 11 in.
8. A cube, 8 in. on a side.
9. How many gallons of water will a cylindrical cistern hold if its diameter is 8 ft. and depth 20 ft.? (See § 110.)
10. The volume of a cylinder is 500 cu. in., and its base has a diameter of 10 in. What is its height?
11. The volume of a prism is 320 cu. ft., and its base is a square, 4 ft. on a side. Find the altitude.
12. A cylinder 25 ft. high has a capacity of 1000 cu. ft. What is the diameter of its base?
13. How many cubic yards of dirt will it take to make a fill 300 yd. long, 10 ft. high, and 30 ft. wide?
14. What must be the diameter of a cylindrical gallon measure 7 in. high?
15. A has a cylindrical cistern, diameter 4', depth 10'. What is the diameter of B's cistern of the same shape and depth, holding 8 times as much as A's?
16. The volume of a cube each edge of which is 6'' is how many times as great as that of a cube each edge of which is 2''?
17. How many square yards of tin will be required to make 42 cylindrical pails, without covers, 10'' in diameter and 12'' deep, allowing 4 sq. ft. for seams and waste on each dozen pails?
18. A rectangular swimming pool is 60 yd. long, 80 ft. wide, and 10 ft. deep. How long will it take to fill it with water to within 3 ft. of the top, if the water runs in at the rate of 8 cu. ft. per second?

PRACTICAL MEASUREMENTS

160. Papering is estimated by the roll, either single roll (24 ft. long by 18 in. wide), or by the double roll (48 ft. long by 18 in. wide). It is more economical to use double rolls on account of the saving in matching the pattern.

NOTE. Imported papers vary in length and width.

161. Use the following rules for estimating the number of rolls required for any given room :

1. Deduct the width of all openings (doors and windows) from the perimeter of the room. Divide this result by the width of the paper to determine the number of strips required for the walls. The length of each strip will be the height of the room.

2. Divide the width of the ceiling (width of the room) by the width of the paper to determine the number of strips required for the ceiling. Each strip will be the length of the room.

3. Determine the number of strips (of each kind) that can be cut from a roll, and divide this into the number of strips required to determine the number of rolls required.

NOTE. Spaces above and under doors and windows (not allowed for in the above calculations) can usually be covered with the strips left over in matching the pattern.

162. Painting and plastering are estimated by the square yard. Make allowance for openings, unless otherwise directed.

NOTE. Contracts usually specify allowances for openings.

163. Carpet is sold by the linear (running) yard ; linoleum by the square yard.

Carpets vary in width ; this, as well as the matching of the pattern, must be taken into account in figuring the number of yards necessary for a given floor. The strips are laid the *long* way of the room.

EXERCISES

1. Find the cost of plastering, papering, and carpeting a room 20 ft. long, 16 ft. wide, and 10 ft. high ($20' \times 16' \times 10'$). It has 5 windows, each $6' \times 3'$, 2 doors $7' \times 3'$, and 1 door $7' \times 6'$. Paper costs $\$1.12\frac{1}{2}$ per double roll ; plastering costs 40¢ per square yard ; the carpet is $\frac{3}{4}$ yd. wide, at $\$2$ per yard.

Plastering :

$$\begin{array}{rcl} \text{Area of walls} & = 2 \times (20 \times 10) + 2 \times (16 \times 10) & = 720 \\ \text{Area of ceiling} & = 20 \times 16 & = 320 \\ \text{Total area} & & \underline{1040} \text{ sq. ft.} \end{array}$$

Openings :

$$\begin{array}{rcl} 5 \text{ windows} & = 5 \times (6 \times 3) & = 90 \\ 2 \text{ doors} & = 2 \times (7 \times 3) & = 42 \\ 1 \text{ door} & = 7 \times 6 & = 42 \\ & & \underline{174} \text{ sq. ft.} \\ \text{Area to be plastered} & & \underline{866} \text{ sq. ft.} \end{array}$$

$\frac{866}{9}$ = number of square yards.

$$\frac{866 \times \$.40}{9} = \$ 38.49, \text{ cost at } 40 \text{¢ per square yard.}$$

Papering :

$$\begin{array}{rcl} \text{Perimeter of room} & = 2 \times (20 + 16) & = 72 \\ \text{Width of 5 windows} & = 5 \times 3 & = 15 \\ \text{Width of 2 doors} & = 2 \times 3 & = 6 \\ \text{Width of 1 door} & = & 6 \\ \text{Total width to be deducted} & \underline{27} \text{ ft.} & \underline{27} \\ & & 45 \text{ ft.} \end{array}$$

$45' \div 18'' (\frac{3}{2}) = 30$ strips, each 10' long, for the walls.

$16' \div \frac{3}{2}' = 10\frac{2}{3}$ or 11 strips, each 20' long, for the ceiling.

As each roll is 48' long, 4 ten-foot strips or 2 twenty-foot strips can be cut from each roll.

Hence, $30 \div 4 = 7\frac{1}{2}$, or 8 rolls for the walls.

$11 \div 2 = 5\frac{1}{2}$, or 6 rolls for the ceiling.

Total required is $8 + 6$ or 14 rolls.

$$14 \times \$1.12\frac{1}{2} = \$15.75.$$

Carpeting :

As the room is 16' wide, as many strips of carpet will be required as $\frac{3}{4}$ yd. (the width of the carpet) is contained in 16'.
 $\frac{3}{4}$ yd. = $2\frac{1}{4}$ '. $16 \div 2\frac{1}{4} = 7\frac{1}{9}$, or 8 strips, each 20' long.

$20 \times 8 =$ number of *feet*; $\frac{20 \times 8}{3} =$ number of *yards*.

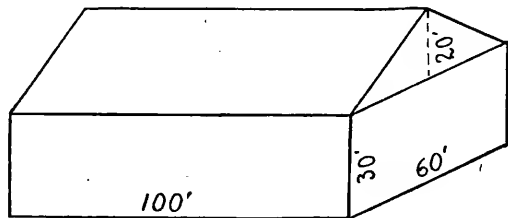
$$\frac{20 \times 8 \times \$2}{3} = \$106.67, \text{ cost.}$$

2. At 20¢ per square yard, how much will it cost to plaster the walls and ceiling of a room $18' \times 15' \times 7' 4''$, making allowance for 2 doors $7' 6'' \times 3' 6''$, and 3 windows $6\frac{1}{2}' \times 3'$?

3. A room is $15' \times 12' \times 9'$. How much will it cost to paper it at $37\frac{1}{2}$ ¢ per double roll, allowing for 2 doors $8' \times 4'$ and for 2 windows $6' 4'' \times 3' 3''$?

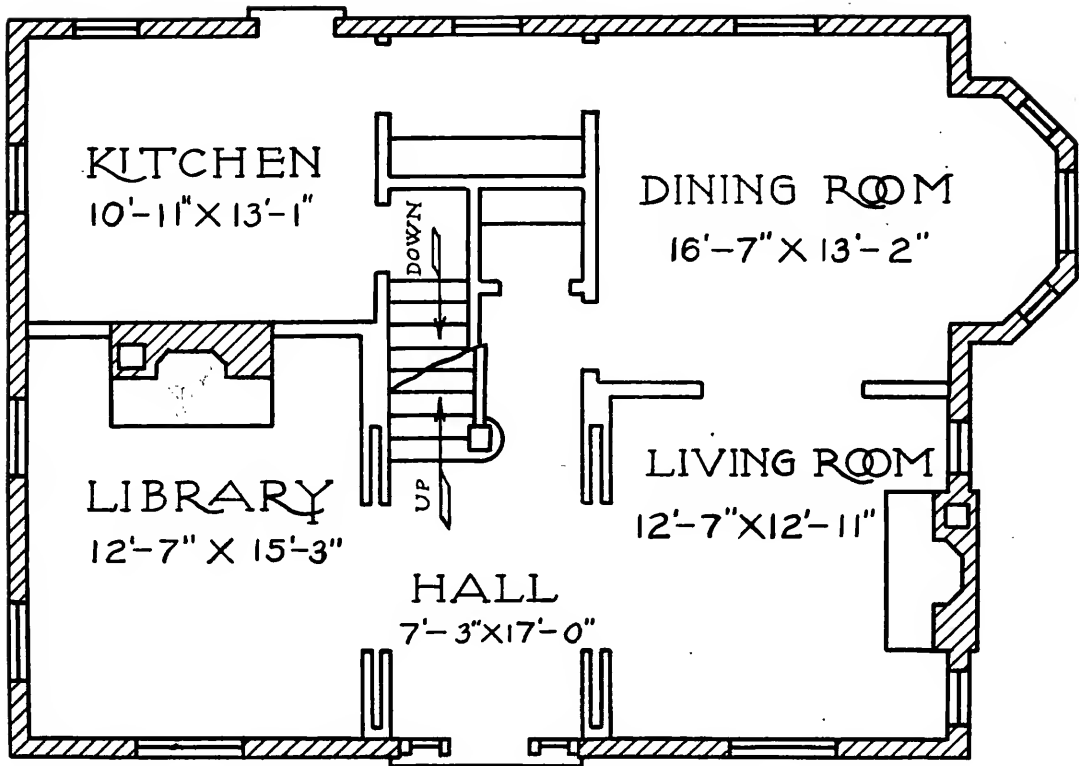
4. A wall is 16' long and 12' high. How many double rolls of paper will it take to paper it, no allowance being made for openings?

5. A barn is $100' \times 60' \times 30'$, with gables 20' above the walls. If no allowance is made for openings, what will it cost to paint it at 5¢ per square yard?



6. How many yards of carpet 1 yd. wide will it take to cover a room $14' \times 10'$, strips running lengthwise?

7. How many yards of carpet $\frac{3}{4}$ yd. wide will it take to cover the library, dining room, and living room, in the



accompanying floor plan, no allowance being made for matching patterns?

8. How many yards of linoleum will it take to cover the kitchen?

9. How much will the carpet in problem 7 cost at \$1.33 per yard?

164. Roofing and flooring are usually measured by the square of 100 square feet. Flooring is sometimes measured by the 1000 board feet.

165. The most common roofing materials are : slate, tiling, shingles, and tin.

The size of slate used is generally 12" x 6", or 24" x 16".

NOTE. Contractors use a table for estimating the amount of slate to be used.

The size of shingles used is generally 16" x 4", or 18" x 4". 16" x 4" shingles are laid with 4" exposed to the weather.

166. Shingles are sold in bundles of 250 each. Allowing for waste, the usual estimate of the number of shingles per square is shown in the following table.

PART EXPOSED TO THE WEATHER	NUMBER SHINGLES PER SQUARE
4 "	1000
$4\frac{1}{2}$ "	900
5 "	800
$5\frac{1}{2}$ "	700

Ex. A gable roof is 40' long and 20' on each side. How many shingles laid 4" to the weather must be bought to cover it?

$$\frac{40 \times 20 \times 2}{100} = 16 \text{ squares.} \quad \frac{\text{length} \times \text{width} \times \text{no. sides}}{\text{no. square ft. in a square}}$$

$$16 \times 1000 = 16000 \text{ shingles (by table).}$$

167. In flooring, a waste of about $\frac{1}{5}$ of the total amount of lumber required is allowed for the "tongue and groove."

Use cancellation when possible :

1. I wish to floor a room 20' \times 16'. How much will the material cost at \$30 per thousand square feet?

2. How many bundles of shingles 16" \times 4" will be required to cover a roof 120' \times 40'?

3. At \$4 per thousand how much will it cost to shingle a roof 100' \times 30', allowing 800 shingles to the square?

4. Find the cost, at \$40 per thousand square feet, of the tongued and grooved flooring for a room 30' \times 20'.

168. Lumber is sold by the 1000 (M) board feet. A board foot is 1 ft. long, 1 ft. wide, and 1 in. thick.

NOTE. Boards less than 1" thick are counted as 1" thick.

169. In making extensions in bills of lumber, point off 3 places and multiply by the price per 1000 feet, using aliquot parts and cancellation.

At \$22 per M what is the cost of 60 pieces of hemlock
 $16' \times 6'' \times 2''$?

$$\frac{16 \times 6 \times 2 \times \overset{5}{\cancel{60}} \times 22}{12} = \$21.12$$

Divide by 12 because the width
 is expressed in inches.

Make the extensions in the following bill of lumber :

200 pieces $18' \times 12'' \times 2''$ — ft.	
500 pieces $14' \times 10'' \times 3''$ — ft.	
150 pieces $16' \times 15'' \times 2''$ — ft.	
250 pieces $12' \times 8'' \times 2''$ — ft.	
400 pieces $16' \times 2' \times 4''$ — ft.	
1000 pieces $16' \times 4'' \times 2''$ — ft.	
Total — ft.	@ \$33.00 per M
600 running feet $2'' \times 4''$	@ 33.33 per M
15000 board feet oak flooring	@ 35.00 per M
20 M shingles	@ 4.50 per M
	Total

170. Wood is measured by the cord. A cord of wood is
 8 ft. long, 4 ft. wide, and 4 ft. high, and contains 128 cu. ft.

NOTE. A cord foot is 1 ft. of the length of such a pile.

Use cancellation :

1. How many cords of wood in a pile $40' \times 20' \times 8'$?

2. At \$5 per cord, how much will it cost to fill with wood
 a shed $25' \times 15' \times 18'$?

171. Stone is measured by the cubic yard, or by the perch
 ($24\frac{3}{4}$ cu. ft.).

172. Bricks are estimated by the M. 22 common bricks
 laid in mortar are counted for each cubic foot of wall.

NOTE. Allowance is always made for openings in ordering material
 and in making contracts.

SPECIAL METHODS

173. To find the number of bricks in a wall, multiply the number of cubic feet in the wall by 22.

174. To find the number of perches of stone, in a wall, divide the number of cubic feet in the wall by $24\frac{3}{4}$.

1. How many perches of stone in a wall $15' \times 8' \times 8'$?

2. How many bricks will be required to erect the four walls of a flat building $60' \times 40' \times 20'$, an allowance of 200 cu. ft. being made for the openings and corners, and the walls being 12" thick?

DIFFERENCES IN TIME

175. Two methods are employed in finding the difference in time between two dates:

1. Exact number of days.

Find the exact number of days from Feb. 2, 1912, to June 4, 1912.

29,	number days in February
<u>2</u>	
27,	number days left in February
31,	number days in March
30,	number days in April
31,	number days in May
<u>4,</u>	number days in June
123	da.

2. Compound subtraction.

What is the difference in time between Oct. 24, 1909, and May 8, 1911?

YR.	Mo.	DA.	
1911	5	8	
1909	10	24	
<u>1</u>	<u>6</u>	<u>14</u>	

May is the 5th month in the year, October the 10th month. The subtraction is performed by the method of denominate numbers. (1 mo. = 30 days)

Find the difference in time between the following dates :

3. From Jan. 23 to Nov. 15 of the same year. (Not a leap year.)
4. From Dec. 13, 1910, to April 3, 1912. (Compound subtraction.)
5. From May 10, 1900, to Jan. 5, 1904. (Compound subtraction.)
6. From Jan. 1, 1913, to Dec. 25, 1913.

APPROXIMATIONS USED IN BUSINESS

176. To find the number of bushels of grain in a bin, multiply the capacity in cubic feet by .8.

177. To find the number of heaped bushels of fruit or vegetables in a bin, multiply the capacity in cubic feet by .63.

178. To find the number of gallons in a cistern or reservoir, multiply the capacity in cubic feet by 7.48.

Find the approximate number of bushels of (a) grain, (b) apples, in the following bins:

- | | |
|--------------------------------|--|
| 1. $12' \times 10' \times 8'$ | 3. $16' \times 8' 4'' \times 7'$ |
| 2. $20' \times 15' \times 10'$ | 4. $18' 3'' \times 15' 9'' \times 12' 4''$ |

Find the approximate number of gallons in the following:

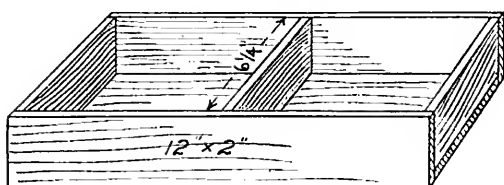
5. A cylindrical cistern, diameter 4', depth 20'.
6. A cylindrical cistern, radius 3', depth 15'.
7. A reservoir, 100 rd. \times 60 rd. \times 50'.

APPLICATION OF PRACTICAL MEASUREMENTS TO MANUAL TRAINING

1. A room $32' \times 28'$ is laid off so as to accommodate 20 pupils in manual training. Allowing one fifth of the floor space for the teacher's use in demonstration work, how much floor space is allowed per pupil?

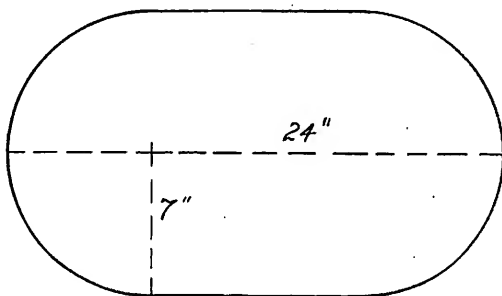
2. Find the total cost of the following manual training tools and supplies:

ARTICLE	PRICE	ARTICLE	PRICE
1 block plane . . .	\$1.50	4 augur bits $\frac{1}{4}$ "', $\frac{1}{2}$ "', $\frac{3}{4}$ "', 1"75
1 jack plane . . .	1.50	1 mallet35
1 rip saw	1.50	4 clamps	8.00
1 mortise saw . . .	1.00	Material for bench	5.00
1 brace75	Miscellaneous	3.65

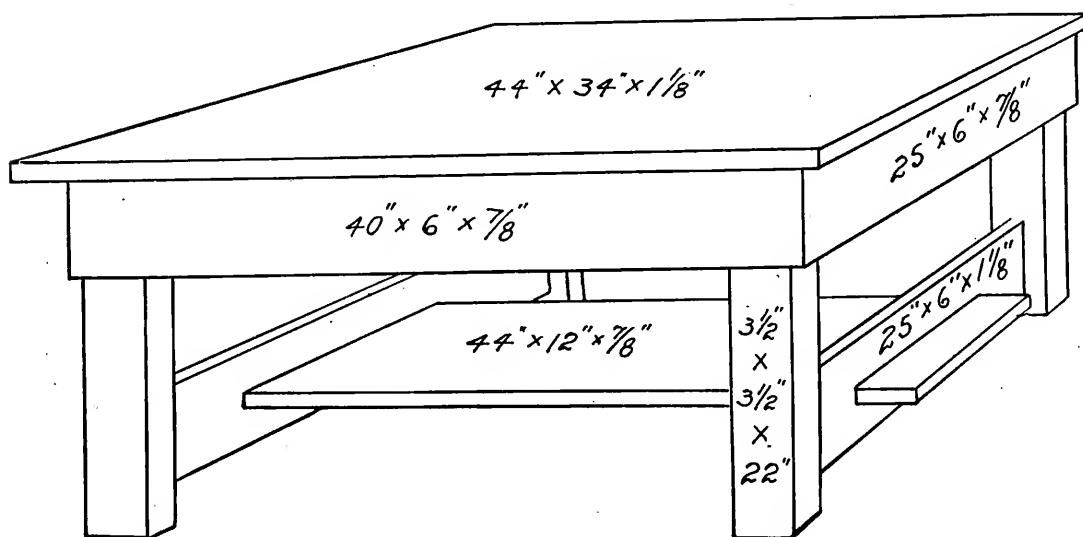


3. At 18 ¢ per board foot, what will poplar lumber $\frac{3}{8}$ " thick cost for making the nail box shown in the illustration?

4. What will the material cost for making the bread board shown in the illustration, using white pine $\frac{3}{8}$ " thick, at 22 ¢ per board foot? Add expense of sanding at 2 ¢ per board foot.



5. At 26 ¢ per board foot, what will the material cost for making the table shown in the illustration? Add expense of sanding at 2 ¢ per board foot for all lumber used.



6. Find the cost of material and sanding in the following, quartered oak being 26 ¢ per board foot, and pine and basswood being 32 ¢; sanding is 2 ¢ per board foot.

MANUAL TRAINING MILL BILL					
<i>Order No.</i> -----					
<i>School</i> H. S. of Commerce			<i>Date</i> 12/4/13		
<i>Name</i> Walter Burkholz			<i>Cut by</i> -----		
<i>Project</i> Music Cabinet			<i>Delivered</i> -----		
<i>Ordered by</i> W. B.			<i>Cutter</i> -----		
No.	KIND OF WOOD	THICKNESS	WIDTH	LENGTH	ARTICLE
1	Quartered oak	1"	18"	21"	Top
2	" "	1"	16"	38"	Sides
1	" "	1"	17"	28"	Door
	Pine	$\frac{3}{8}$ "	24"	24"	
7	Bass	$\frac{3}{8}$ "	$15\frac{1}{2}$ "	17"	Shelves
	All sanded				

MISCELLANEOUS PROBLEMS

GROUP 1

1. Find I in the equation $I = PRT$, if $P = \$300$, $R = .06$, and $T = \frac{1}{4}$.

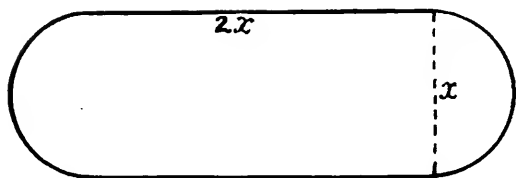
2. Find R in the equation $PRT = I$, if $P = \$250$, $T = \frac{1}{2}$, and $I = \$6.25$.

3. Solve for A in the equation $A = 9\pi$.

4. Find the area of a circle whose radius is 3.

5. Solve for c in the equation $c = 2r$, if $r = 11\frac{1}{2}$.

6. Find the circumference of a circle whose radius is $9\frac{1}{2}$.



7. A race track is laid out as shown in the figure, the ends being semicircles with a diameter x . The sides are $2x$

in length. How long is the track if $x = 184.83$ ft.?

8. Solve for A in the equation $A = 6a^2$, (1) if $a = 2$; (2) if $a = 1$; (3) if $a = .1$.

9. Find the total area of a cube (1) whose edge is 2; (2) whose edge is 1; (3) whose edge is .1.

10. Solve for x in the equation $x = 2a^2$, if $a = 90$ ft.

GROUP 2

1. A baseball diamond is a square 90 ft. on a side. How far is it from home base to second base?

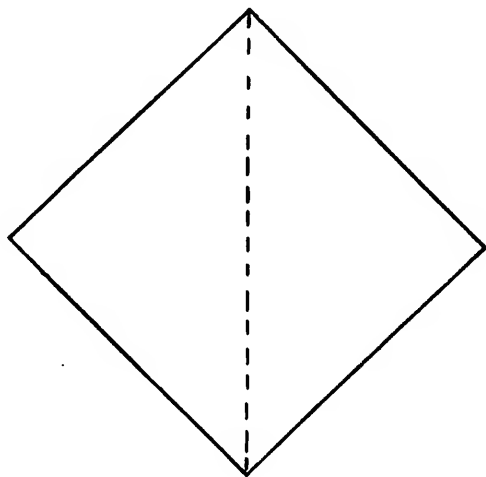
2. The diameter of an automobile tire is 30". How many times does the wheel turn in going $\frac{1}{4}$ of a mile?

3. Two buildings, separated by an alley 15' wide, are 103 and 78 ft. high, respectively. How far is it from the roof of the taller building to the roof of the lower?

4. How many 30" steps must be taken in walking diagonally across a square 40-A. field?

5. How long a rope will it take to reach from the top of a 30-ft. derrick to a point on the ground 18 ft. from its base?

6. A cogwheel railroad gains in elevation 1000 ft. in 1 mi. of track. What horizontal distance has been covered, the track being straight?



7. A railroad track describes a semicircle around the base of the mountain on a radius of 2200 ft. How much longer is the curve than a tunnel straight through would have been?

8. How many gallons of oil can be stored in a cylindrical tank 40' in diameter and 30' high?

9. How many cubic feet of water would cover an acre of ground to a depth of 1 in.?

10. What is the depth of a cylindrical quart measure $4\frac{1}{2}$ " in diameter?

GROUP 3

1. At \$1.28 a yard, how much will it cost to carpet a room $18' \times 16'$ with Brussels carpet $\frac{3}{4}$ of a yard wide?

2. How much will it cost to dig a cellar $30' \times 16' \times 9' 4''$, at $33\frac{1}{3}\phi$ per cubic yard?

3. A grocer bought a 36-gal. barrel of cider for \$5.40, and retailed it so as to gain \$1.80. How much did he receive per quart?

4. What would be the cost of a hardwood floor 2" thick in a room $18' \times 16' 4''$, if the lumber costs $\$33\frac{1}{3}$ per M, and labor costs \$12.75?

5. How much will it cost to paper a room $20' \times 18' \times 10'$, allowing for 2 windows each $5' 4'' \times 3' 6''$ and for 1 double door $8' \times 6'$, if the paper costs 60¢ per double roll?

6. At 32¢ per square yard, how much will it cost to plaster the walls and ceilings in the following dwelling:

Library, $14' 6'' \times 13'$; sitting room, $14' \times 14'$; dining room, $14' \times 10'$; kitchen, $12' \times 8' 6''$; pantry, $6' \times 4'$; 3 bedrooms, respectively, $14' 6'' \times 13'$, $14' \times 12'$ and $10' \times 10'$; a hall, $22' 6'' \times 4' 6''$? All ceilings are 9' high. Allow 600 sq. ft. for openings.

7. How much will it cost to carpet the library, sitting room, and dining room in problem 6 with carpet 1 yd. wide costing $\$1.12\frac{1}{2}$ per yard? (No allowance for matching patterns.)

8. How much will it cost to cover the kitchen and pantry in problem 6 with 72'' linoleum, making no allowance for matching patterns, at 90 ¢ per linear yard?

9. How much will it cost to cover the bedrooms in problem 6 with plain matting 27'' wide at 25 ¢ per yard, and to lay a 36'' runner in the hall at 85 ¢ per yard?

10. How much will it cost to floor a drill hall 80' \times 65' with 1'' hardwood lumber at \$35 per M? (§ 168.)

GROUP 4

1. A man owned the S $\frac{1}{2}$ of NW $\frac{1}{4}$ of a section of land. He bought the NW $\frac{1}{4}$ of the NW $\frac{1}{4}$ of the same section. How many acres did he then have? Draw a diagram showing the location of the land.

2. A river falls 765 ft. in a distance of 35 mi. What is its fall per mile?

3. How many gallons of water will be contained in a cylindrical standpipe 22' in diameter and 72' high?

4. Find the floor space in a six-story building which is 80' \times 30', allowing a space 15' \times 8' on each floor for elevators, and a space 10' \times 4 $\frac{1}{2}$ ' on each floor for stairways.

5. How many cubic yards of material must be moved to excavate a tunnel 8' square and $\frac{1}{8}$ mi. long?

6. Find the approximate number of bushels of apples that can be loaded into a wagon bed 9' \times 3' 4'' \times 32''. (See § 177.)

7. Find the approximate number of bushels of wheat that can be stored in a bin 10' 4'' \times 6' \times 7'.

8. A cylindrical water tank 10' in diameter contains 10575 gal. Find its height. (See § 178.)

9. A field $1320' \times 660'$ is planted to orchard. The rows running the long way of the field are 16' apart, the first and last rows being 18' from the fence; the trees are $32\frac{1}{2}'$ apart in the rows, the outside trees being 26.25' from the fence. How many trees are planted to the acre? (Make diagram.)

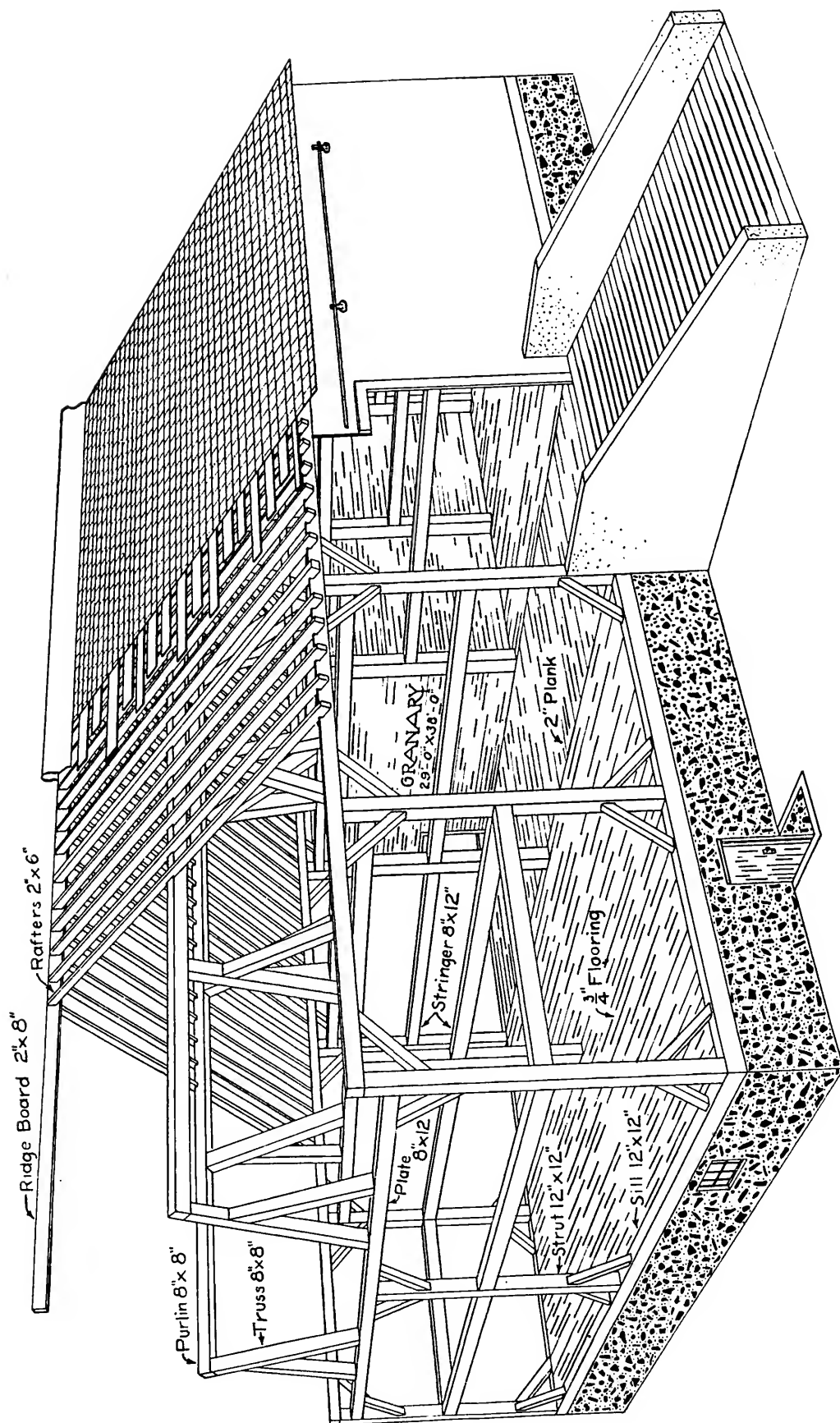
10. A vessel carries 600 T. of wheat to Liverpool, where it is sold at 3s. 15d. per bushel. What is the value of the cargo in United States money?

GROUP 5

The following problems are to be solved with the aid of the illustration on the following page:

DATA

Total length	100'
Total width	60'
Height	
bottom of sill to top of ridge board	40'
bottom of sill to top of plate	20'
Length	
outside struts	18'
inside struts	16'
long members of trusses	11'
short members of trusses	8'
Distance between struts lengthwise of barn	18' except
middle section, which is	22'
Rafters	
over struts $6'' \times 6''$	12 in number
all others $6'' \times 2''$	70 in number
Overhang and extension of roof	18''
Braces at top and foot of struts $6'' \times 4''$	4' 6'' long
Sheathing for roof has 80 rows of boards $4''$ wide and $\frac{3}{4}''$ thick.	
Extension of ridge board, 18'' on each end.	



1. Fill in the number of pieces of each kind in the following:

KIND	NUMBER
Struts 18' long	
Struts 16' long	
Stringers 28½' long	
Stringers 18' long	
Trusses 11' long	
Trusses 8' long	
Braces 4' 6" long	
Rafters 6" × 2"	
Rafters 6" × 6"	

2. If no allowance is made for corners, how many linear feet of each of the following will be needed: sills, plates, purlins, ridge board?

3. What is the length of the rafters, if 18" is allowed for the overhang? (See § 147, 2 c.)

4. Fill in the following, using the table in example 1:

	LINEAR FEET	SIZE	BOARD FEET	REMARKS
Sills	320	12 × 12	3840	No allowance for corners
Struts				
Stringers				
Trusses				
Braces				
Rafters		6 × 6		
Ridge board				
Plates				
Roof sheathing				
Purlins				
Rafters		6 × 2		

5. How many board feet of flooring will be used? How many board feet of plank? (No allowance for anything but sills.)
6. If no allowance is made for openings, how many board feet of $\frac{3}{4}$ " lumber will be used for siding? (For area of gable ends, see § 147, 2 a.)
7. If no allowance is made for openings, how many board feet of $\frac{3}{4}$ " lumber will be needed for the four sides and top of the granary? (Granary 38' \times 29' \times 16'.)
8. Allowing for extension and overhang, find the area of the roof.
9. How many shingles laid 5 $\frac{1}{2}$ " to the weather will be needed?
10. Using all previous data, fill in and extend the following:

	<i>Board Feet</i>
Sills	
Struts	
Stringers	
Trusses	
Rafters	
Braces	
Plates	
Ridge board	
Plank flooring	
Purlins	
<hr/>	
Total	board feet @ \$35.00
Flooring	board feet @ 32.00
Siding	board feet @ 27.50
Sheathing	board feet @ 25.00
Granary material	board feet @ 26.00
— M shingles, per M	<u>5.50</u>
Total	

PERCENTAGE

179. Percentage (per cent = by the hundred) is a commercial method of computing a fractional part, when that fractional part is hundredths.

180. The **sign** (%) per cent, or the decimal fraction showing hundredths, as .25 (25 %), is used.

181. The table of aliquot parts on page 47 will be used for most percentage operations; *e.g.*, $12\frac{1}{2}\%$ or $.12\frac{1}{2}$ of a number is $\frac{1}{8}$ of it.

TO THE TEACHER. Review the table of aliquot parts until the student can recognize all equivalents instantly.

182. The **base** is the number upon which the percentage is computed; *e.g.*, interest is figured on the principal as the base; income upon the investment; etc.

183. The **rate**, or **rate per cent**, is a fractional part of the base to be taken.

184. The **percentage** is the result obtained by multiplying the base by the rate.

185. The **amount** is the sum of the base and the percentage; *e.g.*, principal plus interest equals amount, etc.

186. The **difference** is the remainder when the percentage is subtracted from the base; *e.g.*, cost minus loss equals selling price (remainder).

NOTE. When the rate is a fractional part of 1 %, as .0075 ($\frac{3}{4}\%$), find 1 %, then take the fractional part of it.

EXERCISES

1. What is 25 % of 200 ?
 $\frac{1}{4}$ of 200 = 50.
2. Find $2\frac{1}{2}$ % of 90.
 $2\frac{1}{2}$ % is $\frac{1}{4}$ of 10 %.
 10 % of 90 = 9.
 $\frac{1}{4}$ of 9 = $2\frac{1}{4}$.

Find by inspection:

3. 50 % of 360
4. $12\frac{1}{2}$ % of 24
5. 25 % of 60
6. 20 % of 75
7. $9\frac{1}{11}$ % of 36
8. $37\frac{1}{2}$ % of 32
9. $33\frac{1}{3}$ % of 42
10. 40 % of 43
11. $3\frac{1}{3}$ % of 60
12. $1\frac{1}{4}$ % of 120
13. $1\frac{2}{3}$ % of 360
14. $2\frac{1}{4}$ % of 280
15. $16\frac{2}{3}$ % of 300
16. $\frac{2}{3}$ % of 870

187. Formulas :

Base \times rate = percentage, or $BR = P$.

Percentage \div base = rate, or $\frac{P}{B} = R$.

Percentage \div rate = base, or $\frac{P}{R} = B$.

Base + percentage = amount, or $B + P = A$.

Base - percentage = difference, or $B - P = D$.

EXERCISES

1. Percentage is 400, base 1600, find rate.

$$\frac{P}{B} = R. \quad \text{Then } \frac{400}{1600} = \frac{1}{4} = 25 \%, \text{ rate.}$$

2. Percentage is 300, rate 20 %, find base.

$$300 \div \frac{1}{5} = 1500, \text{ base.}$$

3. A man bought a house for \$5000 and sold it for \$4000. What was his rate of loss?

$$\$5000 - \$4000 = \$1000 \text{ loss.}$$

$$\frac{1000}{5000} = \frac{1}{5}, \text{ or } 20\% \text{ loss.}$$

4. If I sell a house for \$3200, thereby losing 20% of the cost, how much did the house cost?

$$\frac{1}{5} = \text{loss. Then } \frac{4}{5} = \text{selling price.}$$

$$\frac{1}{5} \text{ is } \frac{1}{4} \text{ of } \frac{4}{5}. \text{ Then I lost } \frac{1}{4} \text{ of the selling price.}$$

$$\begin{array}{r} 4 \overline{)3200} \\ \underline{800} \\ 4000 = \text{cost.} \end{array}$$

5. Find 17% of 360.

$$\begin{array}{r} 360 \\ .17 \\ \hline 2520 \\ 360 \\ \hline 61.20 \end{array}$$

Since 17% is not a convenient fractional part of 100%, multiply the base by .17.

Find the missing parts (by inspection when possible):

	BASE	RATE	PERCENTAGE
6.	2400	25	—
7.	1600	20	—
8.	2500	—	500
9.	8000	—	1000
10.	6600	—	2200
11.	—	$12\frac{1}{2}$	3000
12.	—	$44\frac{4}{9}$	1200
13.	30000	—	1000
14.	—	$12\frac{1}{2}$	5000

	BASE	PERCENTAGE	AMOUNT	DIFFERENCE	RATE %
15.	2000	—	2400	—	—
16.	—	1000	6000	—	—
17.	3000	—	—	—	$33\frac{1}{3}$
18.	—	—	4400	—	10
19.	—	—	6000	—	$33\frac{1}{3}$
20.	8000	—	—	2000	—
21.	—	—	14000	—	$8\frac{1}{3}$
22.	—	—	20000	—	$16\frac{2}{3}$
23.	—	—	—	200	$11\frac{1}{9}$
24.	—	1000	—	—	$9\frac{1}{11}$

25. A man bought a house for \$6000. At what price must he sell it to gain 20 % of the cost ?

26. The population of Omaha according to the 1910 census is 124096, a growth of 21 % since the 1900 census. What was its population in 1900 ?

27. A stock of goods damaged by fire sold at 40 % less than cost. If the sale amounted to \$24648, what was the cost ?

28. If 45 gal. of pure vinegar and 5 gal. of water were mixed, what per cent of the whole was pure vinegar ?

29. A grocer mixed 20 lb. of nuts at 10¢ per pound with 30 lb. of nuts at 15¢ per pound. At what price per pound must he sell the mixture to gain 20 % on the cost ?

GAIN AND LOSS

188. The **gross cost** is the original cost plus all expenses.

NOTE. Such expenses might be freight, cartage, import duties, interest on investment to time of sale, insurance, etc.

189. The **net cost** is the original cost.

190. The **net selling price** is the total selling price less all expenses.

NOTE. Such expenses might be advertising, soliciting, discounts for cash, cost of collecting, commissions, storage, etc.

191. In gain and loss :

Cost = base.

Per cent of gain or loss = rate.

Gain or loss = percentage.

Selling price = amount or difference.

192. Formulas :

Cost \times rate = gain or loss, or $BR = P$.

Gain or loss \div rate = cost, or $\frac{P}{R} = B$.

Gain or loss \div cost = rate, or $\frac{P}{B} = R$.

Cost \times (100 % + rate of gain) = selling price.

Cost \times (100 % - rate of loss) = selling price.

Selling price \div (100 % + rate of gain) = cost.

Selling price \div (100 % - rate of loss) = cost.

EXERCISES

1. If a man lost 16 % on his investment by selling a house for \$5000, how much did the house cost him?

Selling price \div (100 % - rate of loss) = cost.

$$\begin{array}{r} \$5952.38, \text{ cost} \\ \hline \text{\text{f}84} \overline{)5000\text{\text{f}00.}} \end{array}$$

NOTE. This method is used when the per cent is not an aliquot part easily handled.

2. Store fixtures costing \$250 were inventoried at the end of the first year at $8\frac{1}{3}$ % below cost. How much was the loss?

3. I sold an 80-A. farm at an advance of 20 %, thereby gaining \$1000. Find the cost per acre.

4. A merchant marks goods 25 % above cost. What is the cost of an article marked at \$30 ?

5. During the first and second years a manufacturer realized gains of 10 % and 20 % on his original investment. During the third year he lost \$1000, which was 20 % of his original capital. How much did he gain the first year ? The second year ?

6. If coffee loses 5 % of its weight in roasting, how much green coffee will be required to produce 1000 lb. of roasted coffee ?

7. A merchant sold a consignment of No. 3 wheat with the understanding that he was to receive $12\frac{1}{2}\%$ of the proceeds. He remitted \$5680 for 6000 bu. How much per bushel did he receive ?

8. I paid \$550.50 freight on an invoice of goods. Later I sold the goods at a profit of 10 % on the entire cost, receiving \$15,695.25. What was the first cost ?

9. On January 1 you begin business, investing \$10000 cash. One year later your resources and liabilities are as follows :

RESOURCES			LIABILITIES		
Cash on hand . .	\$5000		Acc'ts payable . .	\$1200	
Merchandise . .	1500		Notes payable . .	2000	
Real estate . . .	6000				
Office fixtures . .	480				
Acc'ts receivable .	2000				

Find your per cent of gain or loss, based on the investment.

10. A merchant failed in business. The assignee found assets to the amount of \$20000, which would pay 70¢ on the dollar. What was the amount of his indebtedness?

11. A bank with deposits of \$875000 was closed by the bank examiner. Assets of \$750000 were found. What per cent of his money did each depositor receive?

12. I sold a customer goods at a profit of 20% of the cost, but succeeded in collecting only 80¢ on the dollar. What per cent did I gain or lose?

13. A sold goods to B, collecting only 75% of the bill. If A came out even on the deal, what rate of profit, based on the cost, had he added to the cost before selling?

14. A man buys a quarter section of land for \$8500, and sells it in 20-A. farms at an average of \$62 per acre. What per cent of the cost did he gain?

15. Three cement plants are operated by a company. The annual report for 1913 shows the cost of manufacture, selling price, and output of each plant. What per cent of profit per barrel does each plant make on the cost of manufacture, and what per cent of the total annual profit does each plant produce?

PLANT	COST PER BBL.	SELLING PRICE PER BBL.	1913 OUTPUT	ANNUAL PROFIT
A	\$1.10	\$2.20	200000 bbl.	
B	1.00	1.75	500000 bbl.	
C80	1.50	1200000 bbl.	
Total				

COMMERCIAL DISCOUNT

193. Merchandise is usually advertised by manufacturers at **list price** or catalogue price, from which the dealers and jobbers get certain discounts, known as **trade discounts**.

These discounts vary on different kinds of merchandise and according to quantities purchased.

194. Deduction of the trade discount from the list price gives the **net wholesale price**.

NOTE. Many firms which sell at wholesale only, quote a net wholesale price instead of the list price and trade discount.

195. Wholesale dealers and jobbers buy direct from manufacturers at a discount from the net wholesale price because they buy in large quantities. As a result, a series of discounts from the list or catalogue price occurs; in this case the first trade discount of a series is taken from the list price, the next is a discount off the remainder, etc. After all discounts are taken, the remainder is the **net price**.

196. Payment is made in accordance with conditions specified at the time of sale, known as **terms**, which usually require payment of the net amount of the bill within a specified time, as 30 days, 60 days, or 90 days from the **date of shipment** or **date of invoice**. In most cases a discount of from 2% to 6% from the net amount of the bill is allowed for payment in specified time before the bill is due. Thus, a bill that is due in 30 days, when so specified at the time of sale, may be discounted 2% in 10 days from date of shipment. Such terms are usually written "2% 10, net 30" or "2/10, n/30."

197. Discount for payment according to terms is called **cash discount**.

198. Various kinds of merchandise which sell only during certain seasons are often made up and shipped several months ahead, and bills are dated ahead accordingly. For example, holiday goods are often shipped to the jobbers in July with bills dated as "Nov. 1st, 2% 10, net 30." Dress goods for spring and summer wear are often shipped in January and dated as "April 1st, 6% 10, net 60." When a

bill is paid in advance, a discount of $\frac{1}{2}\%$ to 1% per month in excess of other discounts, called **anticipation**, is usually allowed.

199. In commercial discount,

Face of bill = base.

Per cent of discount = rate.

Discount = percentage.

Net amount of bill = difference.

200. Formulas :

Face of bill \times rate = discount, or $BR = P$.

Discount \div rate = face of bill, or $\frac{P}{R} = B$.

Discount \div face of bill = rate, or $\frac{P}{B} = R$.

Face of bill — discount = net amount of bill.

201. Finding the net price.

The first price of a certain article is \$60. If this price is subject to a discount series of 25% and 20% , what is the net selling price?

25% or $\frac{1}{4}$ of \$60 = \$15, first discount.

\$60 — \$15 = \$45, price after the first discount.

20% or $\frac{1}{5}$ off \$45 = \$9, second discount.

\$45 — \$9 = \$36, net selling price.

Find the net selling price (by inspection when possible):

	LIST PRICE	RATE OF DISCOUNT
1.	\$ 300	20%
2.	50	15%
3.	25	10%
4.	75	$33\frac{1}{3}\%$
5.	125	$16\frac{2}{3}\%$
6.	3.75	$12\frac{1}{2}\%$
7.	750	$8\frac{1}{3}\%$
8.	100	40%

9. A buggy listed at \$75 is sold less $33\frac{1}{3}\%$ and 20% . What is the net cost of the purchase?

10. The list price of a certain book is \$.80, subject to a discount of 25% . If the dealer makes 10% , what is his selling price?

11. Find the net amount of the following:

35 boxes oranges at \$6.00

12 doz. cans tomatoes at 96¢

20 bunches bananas at \$1.30

Discounts $12\frac{1}{2}\%$, 5% .

12. Which is the better offer, and how much, on a bill of goods amounting to \$2000, a discount series of 30% , 20% , and 25% , or a series of 50% and $12\frac{1}{2}\%$?

13. The net price of an article is \$2.00. If discounts of 20% and 10% from the list price have been allowed, what is the list price? [Net price \div ($100\% - \text{discount}$) = list price.]

14.

COLUMBUS, OHIO, July 1, 1914.

LOVEMAN, JOSEPH, & LOEB

BIRMINGHAM, ALA.

In account with

The MOREHOUSE MANUFACTURING Co.

2%/10, net 30.

Dated as Nov. 1, anticipation 1% a month.

		$\frac{1}{2}$ gr. Kum Bak Games @ \$48 per gross . . .		
		2 doz. Marathon Racers @ \$12 per dozen . . .		
		$\frac{1}{2}$ doz. 3-way Cars @ \$18 per dozen . . .		
		$\frac{1}{2}$ gr. Hi-Lo games @ \$24 per gross . . .		

What was the amount of the bill July 10? Nov. 1? Dec. 1?

15. What is the net amount of a bill of goods amounting to \$1250, dated Oct. 3, terms $1\frac{1}{2}\%/10$, $1\%/30$, net 60, paid on Oct. 26? What would be the discount if paid on Oct. 8?

16. We bought of Berry Bros. 2000 $\frac{3}{16}'' \times 1''$ roundhead stove bolts at 90¢ per hundred, and 1000 $\frac{3}{16}'' \times 1\frac{3}{4}''$ roundhead stove bolts at \$1.15 per hundred, subject to discounts of 85%, 10%, and 10%. What was the net amount of the bill?

17. An article, listed at \$1.50, is sold by a jobber to a merchant at a net wholesale price of $33\frac{1}{3}\%$ off list. The jobber gets a discount of 25% off the net wholesale price, and the salesman gets a commission of 10% of the price paid by the jobber. If the labor and materials cost the manufacturer 25¢, and his overhead charge is 20% of the net wholesale price, what is the profit to the manufacturer?

18. The F. P. Hall Co. sold the following bill of hardware:

17 gross #8 screws at 87¢ per gross, less $87\frac{1}{2}\%$, 5%, 10%, 5%, $7\frac{1}{2}\%$.

$1\frac{2}{3}$ doz. locks at \$17.50 per dozen, less 50%, 20%, 10%.

$\frac{3}{4}$ doz. locks at \$12.60 per dozen, less $16\frac{2}{3}\%$ and $7\frac{1}{2}\%$.

$3\frac{1}{4}$ doz. $2\frac{1}{2}$ bolts at \$1.25 per dozen, less 80% and 5%.

$2\frac{1}{12}$ doz. bolts at \$3.60 per dozen, less 65%.

What was the net amount of the bill if discounted 2% for cash?

202. To find a single discount equal to a discount series.

In extending a large number of bills from which must be taken the same series of discounts, it is convenient to determine a single discount equal to the quoted discount series.

Find a single rate of discount equivalent to a discount series of 25% and 20%.

SOLUTION 1

1.00 (list price 100%)

.25 ($\frac{1}{4}$ of 100%)

.75

.15 ($\frac{1}{5}$ of 75%)

.60

100% - 60% = 40% single discount.

SOLUTION 2

$$\frac{1}{4} + \frac{1}{5} - \frac{1}{20} = \frac{8}{20} \text{ or } \frac{2}{5}, \frac{2}{5} = 40\%$$

or

$$25\% + 20\% - 5\% = 40\%$$

When the discount series contains but two discounts, from the sum of the discounts subtract their product.

NOTE. — If the discount series has three discounts, find a single per cent equivalent to two of them, and then a single per cent equivalent to the first result and the remaining discount. For example, if the above series had been 25%, 20%, and 10%, it would have been equal to a series of 40% and 10%.

Find a single discount equivalent to the following series of discounts:

1. 10 % and 10 %

2. 25 % and 25 %

3. 20 % and 20 %

4. 40 % and 20 %

5. 30 % and 20 %

6. 50 % and 40 %

7. $33\frac{1}{3}\%$ and $33\frac{1}{3}\%$ 8. $33\frac{1}{3}\%$ and 15 %

9. 20 %, 10 %, and 10 %

10. 40 %, 20 %, and 10 %

11. $16\frac{2}{3}\%$, $66\frac{2}{3}\%$, and 5 %12. 25 %, 10 %, and $33\frac{1}{3}\%$ 13. 25 %, 20 %, and $16\frac{2}{3}\%$ 14. 20 %, 20 %, and $12\frac{1}{2}\%$ 15. $12\frac{1}{2}\%$, $6\frac{1}{4}\%$, and 5 %16. 40 %, 20 %, and $12\frac{1}{2}\%$

17. A firm gives its customers a discount series of 25 %, 20 %, and 10 %. What single discount does it allow?

18. Goods listed at \$300 are sold subject to a discount series of 10 %, 10 %, and 5 %. If the retail merchant sells at a profit of 20 % of the cost, what is his selling price?

MARKING GOODS

203. Merchants often use a private mark to indicate the cost and selling price of goods.

Usually some word or phrase is selected, containing 10 different letters, to be used as a key. These letters represent the 10 digits. When a figure is repeated some letter is used to represent this.

NOTE. Some merchants have different keys for cost and selling prices.

204. Take for a key, "you saw them."

1	2	3	4	5	6	7	8	9	0	Repeater
y	o	u	s	a	w	t	h	e	m	x

Goods which cost \$2.80 and sell for \$3.55, would be marked,

$$\frac{0.50}{1.25}$$

205. To show 20 % profit.

Goods are frequently bought by the dozen and sold by the piece, or bought by the gross and sold by the dozen. It is evident that if 10 articles are sold for the cost of 12, the profit is 2 articles ($\frac{1}{5}$ of 10) or 20 %. Thus, if ties cost \$5.00 per dozen, and 10 ties are sold for \$5.00 (50 ¢ each), the profit will be 20 %.

Hence to make 20 % profit, mark each article at $\frac{1}{10}$ the cost per dozen ; or mark each dozen at $\frac{1}{10}$ the cost per gross. The per cent of profit as computed is based on the cost.

206. To show more than 20 % profit.

If 50 ¢ each for the above ties is a gain of 20 %, it is a selling price of 120 %. To show a profit of 40 %, there must be a selling price of 140 %. This increase of 20 % is $\frac{1}{6}$ of 120 %. Then $\frac{1}{6}$ of 50 ¢ added to 50 ¢ gives 58 $\frac{1}{3}$ ¢ (probably marked at 60 ¢).

207. To show less than 20 % profit.

Subtract the aliquot part.

Mark the cost and selling price of each piece by the above key:

1. Cost \$12 per dozen, sold at a gain of 20 %.
2. Cost \$18.50 per dozen, sold at a profit of 30 %.
3. Boots cost \$26 per dozen, sold at a gain of 25 %.
4. Shoes cost \$21.20 per dozen, sold at a gain of 25 %.
5. Shoes cost \$17.50 per dozen, sold at a gain of 15 %.

6. Pencils cost \$5.00 per gross, sold at a gain of 10 % per dozen.
7. Mark the selling price of towels bought at \$2.90 per dozen and sold at a gain of 55 %.
8. Knives bought at \$3.60 per dozen are sold at 35 % profit. Mark the cost and selling price of each knife.
9. Mark shoes costing \$3.90 per pair so as to gain $33\frac{1}{3}$ %.
10. By selling a hat at \$3.60, I gain 20 %. What was the cost per dozen?
11. A merchant bought boots at \$29.30 per dozen. He sold them at a profit of 20 % after allowing a discount of 10 % from the marked price. What was the marked price?
12. A merchant buys silk marked at \$1.20 per yard, and gets discounts of 25 % and 20 %. Mark the goods so that he will gain 25 %.
13. Caps are listed at \$24 per dozen. The discounts allowed are 25, 20, and $16\frac{2}{3}$ %. Mark a cap so as to gain 60 %.

WHOLESALE AND RETAIL PROFITS

208. In all wholesale and retail businesses, capital is required: (a) for the first cost of merchandise, (b) for the cost of doing business, including selling expenses, advertising, commissions for selling, and the like.

209. **Profit** is the difference between the total cost and the total selling price, which is in excess of the cost.

210. **Per cent of profit** is a per cent of all money invested. Since it is not possible to determine all moneys invested until the goods shall have been sold, and since the cost of doing business, including commissions for selling, must be figured on the selling price, it is advisable to figure both profits and cost of doing business on the selling price.

NOTE. In gain and loss the cost was used as the base, but in problems where cost of doing business must be figured, always use the selling price as the base.

Illustration :

An article cost \$19.50, cartage 50 ¢. If the cost of doing business is 15 %, and a profit of 10 % is to be made, at what price must the article be sold ?

Selling price 100 %
Cost of doing business 15 %
Profit desired 10 %
Wholesale cost $100\% - 25\% = 75\%$ of selling price.
 $\$19.50 + .50 = \20 , cost in money.
 $\$20 \div .75 = \26.67 , selling price.

211. The following Retail Merchants' Table is based upon the selling price. The numbers in the table are per cents.

RETAIL MERCHANTS' TABLE
TABLE FOR FINDING THE SELLING PRICE OF AN ARTICLE

COST OF DOING BUS- NESS	NET PER CENT OF PROFIT DESIRED																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	20	25	30	35	40	50	
15%	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	65	60	55	50	45	35	
16%	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	64	59	54	49	44	34	
17%	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68	63	58	53	48	43	33	
18%	81	80	79	78	77	76	75	74	73	72	71	70	69	68	67	62	57	52	47	42	32	
19%	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	61	56	51	46	41	31	
20%	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	60	55	50	45	40	30	
21%	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	59	54	49	44	39	29	
22%	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	58	53	48	43	38	28	
23%	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	57	52	47	42	37	27	
24%	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61	56	51	46	41	36	26	
25%	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	55	50	45	40	35	25	

RULE. Divide the cost (invoice price with freight added) by the figure in the column of "net per cent profit desired" on the line with per cent of "cost of doing business."

Example :

If a wagon costs	\$60.00
Freight	1.20
	<u>\$61.20</u>
A net profit is desired of	5 per cent
It costs to do business	19 per cent

Take the number in column 5 on a line with 19, which is 76.

$$\begin{array}{r}
 80.526 \text{ (selling price)} \\
 \hline
 76 \overline{)6120.00} \\
 \underline{608} \\
 400 \\
 \underline{380} \\
 200 \\
 \underline{152} \\
 480 \\
 \underline{456}
 \end{array}$$

EXERCISES

1. An automobile cost \$800 at the factory, freight \$18.50; at what price must it be sold to show a profit of 25 % if it costs 20 % to conduct the business ?
2. One gross of brooms cost \$26.50, with freight charges of \$1.50. If it costs 15 % to do business, and the jobber wishes to secure a profit of 15 %, at what price must he sell them per dozen ?
3. A car of coal cost, with freight added, \$108. At what price must it be sold to pay expenses of 21 % and a profit of 7 % ?

4. A piece of farm machinery cost at the factory \$214.50. At what price must it be sold to show a profit of 5 % and cover a selling expense of 17 %?

5. A stove cost \$18, freight \$2. It was marked at a price sufficient to cover expenses of 15 % and show a profit of 35 %. If it was sold at a discount of 10 % from marked price, what was the selling price?

6. A druggist buys toilet articles and rubber goods to the amount of \$100 each. For advertising purposes the toilet articles are sold at cost. If the selling expense of both is 15 %, at what per cent profit must the rubber goods be sold to realize 10 % on the entire transaction?

7. Extend and foot the following bill:

DAYTON, OHIO, Jan. 16, 1914.

THE KROGER COMPANY,
COLUMBUS, OHIO.

In account with

The E. C. HARLEY COMPANY.

200 lb.	No. 6 coffee @ 30¢					
100 lb.	Dayton roast coffee @ 25¢					
50	boxes laundry soap @ 95¢					
20 lb.	currants @ 12¢					
200	doz. canned goods @ \$1.25					
200	doz. canned goods @ 95¢					

At what price must each article be sold to realize a profit of 12 % if the expense of doing business is 18 %?

COMMISSION

212. Commission or brokerage is the sum charged by an agent for transacting business for another.

The agent is called the **broker, commission merchant, or collector**, according to the character of the business trans-

acted. The one for whom the business is transacted is called the **principal**.

213. A **guaranty** is an additional charge made by the agent for assuming the risk on credit sales, or for the quantity and quality of goods.

214. Merchandise sent by a principal (**shipper** or **consignor**) is called a **shipment**.

215. Merchandise received by an agent (**consignee**) is called a **consignment**.

216. **Gross proceeds** is the total amount received by the agent for the goods.

217. **Net proceeds** is the amount remaining after all expenses have been deducted.

218. The **prime cost** is the actual cost of the merchandise.

219. The **gross cost** is the total cost, including expenses, etc.

220. An **account sales** or **account purchase** is a statement itemizing the entire transaction.

221. Apply the principles of percentage.

Gross proceeds or prime cost = base.

Rate of commission = rate.

Commission = percentage.

Gross cost or gross proceeds = amount.

Net proceeds = difference.

Formulas :

Gross proceeds or prime cost \times rate of commission = commission, or

$$BR = P.$$

Commission \div rate = gross proceeds or prime cost, or

$$\frac{P}{R} = B.$$

Commission \div gross proceeds or prime cost = rate, or

$$\frac{P}{B} = R.$$

Prime cost + commission = gross cost.

Gross proceeds - commission = net proceeds.

EXERCISES

Supply the missing parts (by inspection):

	GROSS PROCEEDS	RATE OF COMMISSION	COMMISSION	NET PROCEEDS
1.	\$4500	$2\frac{1}{2}$	—	—
2.	1800	$12\frac{1}{2}$	—	—
3.	1200	$6\frac{2}{3}$	—	—
4.	1250	3	—	—
5.	1600	$8\frac{1}{3}$	—	—
6.	1800	$16\frac{2}{3}$	—	—
7.	7500	—	1500	—
8.	1400	—	400	—
9.	1500	—	600	—
10.	1800	—	400	—

Supply the missing part:

	GROSS PROCEEDS	RATE OF COMMISSION	RATE OF GUARANTY	NET PROCEEDS
11.	\$ 4500	$2\frac{1}{2}$	2	—
12.	1800	10	—	1530
13.	12000	—	$2\frac{1}{2}$	9700
14.	1350	5	1	—
15.	1200	$12\frac{1}{2}$	—	—

16.

ACCOUNT SALES

20000 lb. of coal	\$4	per ton	—	—		
16000 lb. of coal	\$3.50	per ton	—	—		
40000 lb. of hay	\$12	per ton	—	—		
60 T. of hay	\$10	per ton	—	—		
3000 lb. of wheat	70¢	per bu.	—	—		
7000 lb. of oats	60¢	per bu.	—	—		
1500 lb. of corn (shelled)	80¢	per bu.	—	—	—	—
CHARGES						
Freight, \$250; storage, \$135; drayage, \$60; insurance, \$10.			—	—		
Commission, 5%			—	—	—	—
Net Proceeds					—	—

17. An agent's commissions were \$2500 in one month. If he transacted business on a 5% commission, how much business did he handle that month?

18. An agent sold 40000 lb. of wheat. After deducting his commission of 2%, he sent his principal \$490. For how much per bushel was the wheat sold?

19. A manufacturer remits his agent \$3150 with which to buy cotton. How much does the agent invest if his commission is 5%?

$$[\text{Amount} \div (1 + \text{rate}) = \text{investment}.]$$

20. A commission merchant remits \$8000 to invest in wheat, after all expenses are paid. Charges were as follows: guaranty, 2%; drayage, \$10; freight, \$100; commission, $2\frac{1}{2}\%$. Find the commission, and the amount actually invested in wheat.

21. A salesman for a wholesale house works on a commission of 5%. On sales in excess of \$100000 per year he gets an added 1%. From the following report of his sales, figure his total commissions, and his net income above expenses.

MONTH	SALES	EXPENSES
Jan.	\$21673.85	\$521.60
Feb.	27421.96	608.35
March	28316.40	560.25
July	18629.90	389.40
August	23718.06	465.75
Sept.	30163.45	720.80
Totals		

22. A salesman is guaranteed a salary of \$2000 per year, with the understanding that he is to receive not less than 4% of his total sales. What is his income in cash in the following years? What per cent of his sales for 1910 did he receive as salary? What per cent of his sales for the three years is his income for that time?

YEAR	SALES	INCOME
1910	\$41675.00	—
1911	52317.81	—
1912	64500.28	—
Totals		

MISCELLANEOUS PROBLEMS

GROUP 1

1. Give the fractional equivalent for the following percentages: 5%, $6\frac{2}{3}\%$, $8\frac{1}{3}\%$, 10%, $11\frac{1}{9}\%$, $12\frac{1}{2}\%$, 20%, 25%, $33\frac{1}{3}\%$, 50%, $62\frac{1}{2}\%$, 75%.

2. If $B=200$ and $R=.05$, find P in the equation $BR=P$.

3. If $B=600$ and $P=200$, find R in the equation $BR=P$.

4. A man bought a drug store for \$ 6000 ; he gave a mortgage for 60 % of it. What was the amount of the mortgage ?

5. A merchant failing in business was able to pay 85 % of his debts. How much did he pay A, if he owed him \$ 4000 ?

6. A man's income is \$1500 per year. His expenses are as follows : rent, \$ 300 ; household, \$ 500 ; heat, \$ 50 ; light, \$ 20 ; clothing, \$ 200 ; miscellaneous, \$100. What per cent of his salary does he save ?

7. A town lot is valued at \$ 2000, caused by a 25 % rise in the value of real estate. If the owner valued it before the rise at 20 % more than he paid for it, what did it cost him ?

8. A house is sold for \$ 10000, at a loss of 20 % of the cost. Had it been sold for \$ 15000, what would have been the per cent of gain ?

9. A man bought a farm for \$ 6000. Disregarding interest, he can pay for the property in 12 years by saving 20 % of his income. What is his income ?

10. A merchant's cost of doing business is $33\frac{1}{3}$ % of his gross sales ; his profits are 20 % of his gross sales. If the gross sales for the year 1912 are \$ 50000, what per cent of his profits is his cost of doing business ?

GROUP 2

In applying the principles of percentage to the solution of problems, as in Gain and Loss, Commission, etc., pick out *B*, *R*, and *P* before attempting to solve.

1. Supply the missing parts :

COST	SELLING PRICE	GAIN	LOSS	RATE
\$ 2500	\$ 3000	—		—
\$ 5000	—		\$ 500	—
—	\$ 6000	—		20 %
—	\$ 15000		—	$6\frac{1}{4}$ %
—	—	\$ 2000		20 %

2. An article is sold for 50¢, at a profit of 15¢. What formula can be applied to the solution of this problem? What two parts are given, and what is to be found?

3. Make up a problem, using the formula $\frac{P}{R} = B$.

4. A bought 50 hogs for \$325; he sold them so as to gain $16\frac{2}{3}\%$ on the cost. How much did he receive for each hog?

5. A trunk is manufactured at a cost of \$4.75. It is sold at a profit of 40% of the cost by an agent who charged a commission of 10%. How much did the manufacturer get for it?

6. A clothing merchant sold a suit of clothes for \$25. If he gained $33\frac{1}{3}\%$ on the cost, what was the cost of the suit?

7. An automobile which cost, when new, \$1350, was sold a year later for \$837.50. What was the per cent of loss?

8. A manufacturer gained \$15000 in 1911, and lost \$6700 in 1912. If the capitalization was \$75000, what was the per cent of gain for the two years?

9. A commission merchant received a shipment of 6000 bu. of potatoes. He sold $16\frac{2}{3}\%$ of them at \$1.25 per bushel; $33\frac{1}{3}\%$ at \$1.12 $\frac{1}{2}$; 40% at \$1.10; the remainder at cost. If they cost him \$1 per bushel, how much did he gain?

10. A bought a stock of merchandise for \$12000. He sold it at a gain of 20% of the cost, but was unable to collect 20% of the bill. What per cent of the cost did he make on the deal?

GROUP 3

1. Find the missing parts :

LIST PRICE	NET PRICE	DISCOUNT	RATE OF DISCOUNT
\$300	—	\$18	—
—	\$250	\$50	—
—	—	50¢	$2\frac{1}{2}\%$
—	\$75	—	$16\frac{2}{3}\%$
87¢	—	—	7%

2. A dealer can buy the same line of goods from either of two wholesale houses. The first offers him discounts of 20%, 15%, and 5%; the second offers 25%, 10%, and 5%. Which offer is the better, and how much?

3. A gas company offers a discount of 5¢ per thousand cubic feet off the contract price of 35¢ per thousand cubic feet on bills paid before the 10th of each month. What per cent of each bill is saved by taking advantage of the offer?

4. Which is greater, a discount series of 15%, 20%, and 5%, or a straight discount of 40%?

5. A merchant bought a bill of goods on terms 2%/10, net 30. He paid the bill 5 da. after its date, thereby saving \$17. What was the face of the bill?

6. A wholesaler sold ten bills of goods to customers, allowing on each discounts of 25%, 10%, and 5%. What were the discounts, if the bills were as follows: \$135.68, \$320, \$56.70, \$301, \$827.32, \$26.81, \$443.49, \$72.50, \$625, \$281.33?

7. The net proceeds of a bill on which discounts of 10% and 5% had been allowed were \$256.50. What was the face of the bill?

8. An article listed at \$1.20 cost a dealer 72¢. If the first discount was 20%, what was the second discount?

9. Goods listed at \$50 are discounted 40% and $33\frac{1}{3}\%$ to a retailer. What discount must the retailer give off the list price in addition to 40% in order to gain 25% of the cost?

10. What is the net discount of the following bill, if paid on April 1? What would it have been, if paid March 1, anticipation being $1\frac{1}{2}\%$ per month?

Feb. 1, 1914.

O. J. MILLER,
MT. GILEAD, OHIO.

In account with

BURLEY & STEVENS, Inc.

2/30, net 60

Date April 1

18	pr. Boys' grade # 8134 @ \$2.15				
18	pr. Boys' grade # 8163 @ \$2.40				
16	pr. Boys' grade # 9143 @ \$1.85				
20	pr. Boys' grade # 8163 G @ \$2.25				
12	pr. Boys' grade # 8143 a @ \$1.75				

GROUP 4

1. Using the key in § 204, mark shoes costing \$2.40 a pair so as to gain $66\frac{2}{3}\%$.

2. The following price tag was put on a sweater: $\frac{u.oa}{a.xm}$.

What per cent of the cost is the profit on the sweater? (Use key in § 204.)

3. Make up a key for marking goods similar to the one in § 204. Make a price tag showing cost per piece on an article costing \$24 per dozen, and a selling price per piece to gain 30%. (§ 206.)

4. A suit of clothes costing \$6.50 is marked to sell at \$10, but is sold for \$9.87. What per cent of the cost is sacrificed by selling below the marked price?

5. A merchant marked an article costing 50¢ to sell at 75¢, thinking he was making a net profit of 50%. If his cost of doing business is 20%, at what should he have sold it in order to make actually 50% profit?

6. Property costing \$4800 is sold to net the owner 20% profit after paying a commission for selling of 5%. For what price does it sell? (§ 211.)

7. The capital employed in a state's industries is \$1,300,732,732, and the expense of operating is \$1,282,845,514. What per cent of the capital is the operating expense?

8. A farmer can buy machinery at the manufactory for \$138, less 3% for cash, and it will cost him \$12 freight charges to ship it to his home. Is it better to buy at this price, or at \$150 delivered at his home, less 3% for cash?

9. An agent purchased 50 bales of cotton at an average of \$60.10 per bale. What is his commission at $1\frac{2}{3}\%$?

10. A real estate firm charges, as commission, 5% of the first \$2500 involved in any transaction, and 2% of any amount over the first \$2500. What will it receive for selling property for \$4000? for \$5300? for \$1625? for \$108000? for \$17783.60?

GROUP 5

1. A man bought 1000 bu. of wheat at \$1.01 $\frac{1}{2}$ per bushel, and 5000 bu. of corn at 44 $\frac{1}{2}$ ¢ per bushel. He sold the wheat for \$1.02 $\frac{5}{8}$ per bushel, and the corn at 43 $\frac{7}{8}$ ¢ per bushel. How much was his gain or loss?

2. An apartment building cost the owner \$28000, and he estimates his yearly expenses as follows: taxes, \$310; insurance, \$60; water, \$70; janitor, \$300; fuel, \$460; repairs, \$120. What yearly rental must he get from each of 12 apartments to net him 6% on his investment?

3. A creditor agrees to settle a debt at 90¢ on the dollar. He receives \$5327.38, after paying a commission of 2% for collecting. What was the amount of the debt?

4. A commission merchant receives \$42.18 in payment of a commission of 2% and expenses of \$3.10. What was the amount of the sale?

5. A life insurance agent receives 30 % of all premiums on new business. If the average premium paid on his new business is \$31.50, how many premiums will he have to secure to earn an income of \$1956.15?

6. For what must property be sold to net the owner \$8730 after paying a commission of 3 % for selling?

7. Property selling for \$8250 pays the agent negotiating the sale $3\frac{1}{2}$ % commission. If it nets the owner $12\frac{1}{2}$ % profit on the cost, how much did it cost?

8. Property costing \$3100 sells for \$3750. If the selling agent receives a 3 % commission, what per cent of the gross profit goes to the selling agent?

9. A retail merchant takes in \$800 a month at an expense of \$200 a month. At what must he sell an article costing him 25¢ in order to net 15 %?

10. What is the total amount of commissions received by a professional shopper in a large city, who purchases the following goods:

BOUGHT FOR	ARTICLE AND PRICE	AMOUNT		RATE	COM.	
Mrs. A	12 yd. silk @ \$1.25			5 %		
	8 yd. lace @ 87 ¢					
Mrs. B	1 brass bed, \$32.50			1 %		
	1 dresser, \$21.75					
Mrs. C	6 pr. gloves @ \$1.35			5 %		
	6 pr. hose @ 85 ¢					
	2 doz. hdkfs. @ 35 ¢ each					
Mrs. D	1 umbrella @ \$5.50			6 %		

INTEREST

222. Interest is an allowance made for the use of money.

223. Three factors enter into interest calculations, viz., principal, rate per cent, and time.

224. Certain per cents are allowed by law. The legal rate ranges from 5% to 8% in the different states.

Any charge above the rate allowed by law is called **usury**, and is prohibited under various penalties in different states.

NOTE. Debts of all kinds are subject to interest from the time they become due, but not before, unless specified.

225. Interest is commonly computed on 360 da. to the year, 30 da. to the month, and 12 mo. to the year. This is called **common interest**. Time less than one year is found by counting the actual number of days. Long periods of time are found by compound subtractions. Rate is by the year, unless otherwise specified.

226. Interest formulas :

Principal \times rate \times time (in years) = interest, or $PRT = I$.

Interest \div (rate \times time) = principal, or $\frac{I}{RT} = P$.

Interest \div (principal \times time) = rate, or $\frac{I}{PT} = R$.

Interest \div (principal \times rate) = time, or $\frac{I}{PR} = T$.

Principal + interest = amount, or $P + PRT = A$.

227. (a) Find the interest on \$200 for 4 mo. at 6%.
Find the amount.

$PRT = I$, where $P = \$200$, $R = .06$, and $T = \frac{1}{3}$.

Substituting these values in the equation, we have

$$\$200 \times .06 \times \frac{1}{3} = \$4, \text{ interest.}$$

$$\$200 + \$4 = \$204, \text{ amount.}$$

(b) What principal will amount to \$240 in 4 yr. at 5%?

$$P + PRT = A.$$

Factoring, $P(1 + RT) = A$, $P = \frac{A}{1 + RT}$.

Substituting the values $P = \frac{240}{1 + .2}$.

Solving, $240 \div 1.2 = \$200$, principal.

228. Supply the missing parts :

	PRINCIPAL	RATE	TIME	INTEREST	AMOUNT
1.	\$75	6%	3 yr.	—	—
2.	\$150	4%	1½ yr.	—	—
3.	\$125	—	1 yr.	\$6.25	—
4.	\$300	7%	—	\$84	—
5.	—	5%	3 yr.	\$75	—
6.	—	4½%	4 yr.	—	\$1180
7.	\$1200	—	2 yr. 6 mo.	\$105	—
8.	\$1000	5%	1 yr. 4 mo.	—	—

229. Cancellation method.

Find the interest on \$350.50 for 1 yr. 2 mo. 11 da. at 6%.

$$\$350.50 \times \frac{6}{100} \times \frac{431}{360} = \text{int.}$$

$$\frac{\$350.50 \times 431 \times \cancel{6}}{100 \times \cancel{360}} = \frac{\$151065.50}{6000}$$

$$60 \quad \$151.0655 \div 6 = \$25.18 \text{ int.}$$

1 yr. 2 mo. 11 da. = 431 da.; this is reduced to years by dividing by 360. In applying the formula $PRT = I$, cancel 6 into 360. In dividing by 6000, move the decimal point 3 places to the left and divide by 6. Hence, to find the interest at 6%, multiply the principal by the number of days, move the decimal point 3 places to the left, and divide by 6.

Find the interest on \$ 7563 at 6 % for 197 da.

$$\begin{array}{r}
 \cancel{7563} \\
 \begin{array}{r}
 \cancel{6} \overline{) 2521} \\
 2 \overline{) 197} \\
 \hline
 17647 \\
 22689 \\
 2521 \\
 \hline
 496637 \\
 \hline
 248.3185
 \end{array}
 \end{array}$$

The form here used is the most convenient for canceling and performing the other necessary operations. Cancel 3 into 7563 and into 6; multiply 2521 by 197 without recopying; divide the result by 2, and point off 3 places in the result.

\$ 248.32, int.

230. To find the interest at any other rate than 6 %. Add or subtract the necessary aliquot part to the interest at 6 %.

If the rate in the above problem had been 5 %, find the interest.

$$\begin{array}{l}
 6) \$ 248.3185 \\
 \hline
 41.3864 = \text{int. at } 1 \% \text{ (1 \% is } \frac{1}{6} \text{ of } 6 \% \text{).} \\
 \$ 206.93 = \text{int. at } 5 \% \text{ (Subtraction).}
 \end{array}$$

NOTE. In all problems involving dollars and cents, carry the decimal places to the final result, then if the figure in mills is 5 or more, add 1 to the cents in the result; if less than 5, drop it. Many mistakes in bookkeeping and office work can be avoided by observing this rule.

Find the interest :

	PRINCIPAL	RATE %	TIME
1.	\$ 300	6	3 yr. 3 mo. 5 da.
2.	450	4	7 mo. 6 da.
3.	248.25	5	2 yr. 9 mo.
4.	796.50	6	1 yr. 3 mo. 15 da.
5.	503.40	3	90 da.
6.	872	8	93 da.
7.	897.50	6	143 da.
8.	15000	5	3 yr. 5 mo. 17 da.
9.	5000	6	60 da.
10.	9000	4	35 da.

231. 60-day method.

60 days is $\frac{1}{6}$ of one year (360 da.), $\frac{1}{6}$ of 6 % = 1 %. The interest for 60 da. at 6 % is therefore 1 % of the principal.

The interest on \$50 at 6 % for 60 da. would be \$0.50; therefore, *to find the interest for 60 da. at 6 %, point off two places to the left of the decimal point.*

232. If the per cent is other than 6 %, find the proper aliquot part of the interest at 6 %.

233. Time. If the time is other than 60 da., add or subtract the proper aliquot part of 60 da.

Find the interest on \$750 at 5 % for 80 da.

$$\begin{array}{r}
 \$ 7.50 = \text{interest for 60 da. at 6 \%} \\
 2.50 = \text{interest for 20 da. at 6 \%} \\
 \hline
 6) \$ 10.00 = \text{interest for 80 da. at 6 \%} \\
 1.67 = \text{interest for 80 da. at 1 \%} \\
 \hline
 \$ 8.33 = \text{interest for 80 da. at 5 \%}
 \end{array}$$

Find the interest :

	PRINCIPAL	RATE %	TIME
1.	\$ 98	6	60 da.
2.	150	7	60 da.
3.	240	8	90 da.
4.	1450	$4\frac{1}{2}$ ($\frac{1}{4}$ less than 6 %)	72 da.
5.	2380	$6\frac{1}{2}$	33 da.
6.	868.40	4	120 da.
7.	1000	$3\frac{1}{2}$	75 da.
8.	350.75	6	54 da.
9.	790	4	1 mo. 20 da.
10.	575.34	5	2 mo. 18 da.

234. Six per cent method.

The interest on \$1 at 6% for 1 yr. is 6¢.

Then it will be 1¢ for 60 da., and 1 mill for 6 da. Why?

Find the interest on \$2340.20 at 7% for 2 yr. 2 mo. 15 da.

\$0.12 = int. on \$1 for 2 yr. at 6%.

.01 = int. on \$1 for 2 mo. at 6% (2 mo. = $\frac{1}{12}$ of 2 yr.).

.0025 = int. on \$1 for 15 da. at 6% (15 da. = $\frac{1}{4}$ of 2 mo.).

\$0.1325 = int. on \$1 for 2 yr. 2 mo. 15 da. at 6%.

2340.20

265 00

5 300

39 75

265 0

6)310.076500 = int. on \$2340.20 at 6%.

51.6794 = int. on \$2340.20 at 1%.

\$361.7559 = \$361.76, int. at 7%.

Find the interest and amount:

	PRINCIPAL	RATE %	TIME
1.	\$758.12	6	1 yr. 2 mo. 12 da.
2.	896.50	6	2 yr. 4 mo. 15 da.
3.	3500.	4	3 yr. 4 mo. 18 da.
4.	450.75	5	6 mo. 24 da.
5.	185.54	6	1 yr. 5 mo. 6 da.
6.	1000.	8	4 yr. 3 mo. 19 da.
7.	3428.	6	from Apr. 3 to July 25.
8.	26250.	3	from 5/20/09 to 11/24/12.
9.	7368.58	2	from 2/10/04 to 5/18/06.
10.	596.33	$3\frac{1}{2}$	from 1/10/11 to 12/21/12.

235. Simple interest table.

The accompanying table shows the interest on \$1000 from 1 da. to 1 mo., and 1 mo. to 6 mo. at 3% to 7%. For instance, \$1000 will earn, in 20 da., \$2.22 at 4%, and \$2.77 at 5%. In 2 mo., 11 da., it will earn, at 7%, \$11.66 + \$2.13, or \$13.79.

236. To find what \$100, \$750, or any other aliquot part of \$1000 will earn, determine the amount for \$1000 from the table, and take the necessary aliquot part of it. Thus: What will \$275 earn in 5 mo. and 21 da. at 6%?

DAYS	3%	4%	5%	6%	7%
1	\$.08	\$.11	\$.13	\$.16	\$.19
2	.16	.22	.27	.33	.38
3	.25	.33	.41	.50	.58
4	.33	.44	.55	.66	.77
5	.41	.55	.69	.83	.97
6	.50	.66	.83	1.00	1.16
7	.58	.77	.97	1.16	1.36
8	.66	.88	1.11	1.33	1.55
9	.75	1.00	1.25	1.50	1.75
10	.83	1.11	1.38	1.66	1.94
11	.91	1.22	1.52	1.83	2.13
12	1.00	1.33	1.66	2.00	2.33
13	1.08	1.44	1.80	2.16	2.52
14	1.16	1.55	1.94	2.33	2.72
15	1.25	1.66	2.08	2.50	2.91
16	1.33	1.77	2.22	2.66	3.11
17	1.41	1.88	2.36	2.83	3.30
18	1.50	2.00	2.50	3.00	3.50
19	1.58	2.11	2.63	3.16	3.69
20	1.66	2.22	2.77	3.33	3.88
21	1.75	2.33	2.91	3.50	4.08
22	1.83	2.44	3.05	3.66	4.27
23	1.91	2.55	3.19	3.83	4.47
24	2.00	2.66	3.33	4.00	4.66
25	2.08	2.77	3.47	4.16	4.86
26	2.16	2.88	3.61	4.33	5.05
27	2.25	3.00	3.75	4.50	5.25
28	2.33	3.11	3.88	4.66	5.44
29	2.41	3.22	4.02	4.83	5.63
MONTHS					
1	2.50	3.33	4.16	5.00	5.83
2	5.00	6.66	8.33	10.00	11.66
3	7.50	10.00	12.50	15.00	17.50
4	10.00	13.33	16.66	20.00	23.33
5	12.50	16.66	20.83	25.00	29.16
6	15.00	20.00	25.00	30.00	35.00

\$1000 earns \$28.50.

\$ 250 earns $\frac{1}{4}$ of this,
or \$7.125.

\$ 25 earns $\frac{1}{10}$ of \$7.125, or \$.7125.

\$ 275 earns \$7.125 + \$.7125, or \$7.84.

EXERCISES

Find the interest on :

1. \$300 for 18 da. at 5 %.
2. \$125 for 1 mo. 10 da. at 4 %.
3. \$260 for 3 mo. 5 da. at 7 %.
4. \$55 for 2 mo. at 6 %.
5. \$100 for 1 mo. at 2 %. (2 % is $\frac{1}{2}$ of 4 %.)
6. \$1000 for 15 da. at $\frac{1}{2}$ %.
7. \$1000 for 23 da. at $3\frac{1}{2}$ %.
8. \$1500 for 4 mo. 19 da. at $4\frac{1}{2}$ %.
9. \$825 for 6 mo. 21 da. at $5\frac{1}{2}$ %.
10. \$33.33 for 5 mo. 17 da. at $6\frac{1}{2}$ %.
11. How much does a man lose if he allows \$2000 to lie idle for 30 da., when money is worth 5 %?
12. What is the earning power of \$100 at 6 % for 8 mo. 15 da.?
13. If \$1800 lies idle for 1 mo. 20 da., what is the loss at $4\frac{1}{2}$ %?
14. A man refuses to lend \$50 at 7 % for 4 mo. What amount of interest does he sacrifice in refusing?
15. A depositor allows \$3725 to remain in the bank from May 4 to Nov. 4. If he had transferred the money to a savings account at $3\frac{1}{2}$ %, how much would he have gained in interest?
16. Find the interest on \$3000 at 6 % from Nov. 15, 1912, to May 10, 1913.
17. In Ohio, a Loan Co. may deduct 10 % of the loan and charge 8 % interest on the total amount. How much interest is received on a \$100 loan for 4 months?

SPECIAL INTEREST METHODS

237. (a) Find the interest on \$854.30 for 93 da. at 8%.

$$\begin{array}{r}
 \$854.30 \\
 \cancel{36} \quad \cancel{93} \quad 31 \\
 \cancel{39} \quad \cancel{8} \quad 2 \\
 \hline
 62(2 \times 31) \\
 1\ 70860 \\
 51\ 2580 \\
 3)52.96660 \\
 \$17.655, \text{ int.}
 \end{array}$$

Principal \times time (in years) \times rate = interest. As the time is expressed in days, divide this product by 360 (by 36 and move the decimal point 1 place to the left). Hence, divide the product of the principal, the number of days, and the rate, by 360.

Notice that the rate is *per cent* in pointing off decimal places in the result. In the illustration, point off 5 places, 2 for the cents in the principal, 2 for the hundredths in the rate, and 1 for canceling the 0 in 360.

(b) Find the interest on \$658 at 5% for 6 mo.

$$\begin{array}{r}
 4)\$65.80 \\
 \$16.45, \text{ int.}
 \end{array}$$

The interest at 5% for 6 mo. is the same as the interest at 10% for 3 mo. Hence, to find the interest at 5%, point off 1 place to the left, and find the aliquot part $\frac{1}{2}$ the time is of one yr.

(c) Find the interest on \$300 at 8% for 6 mo.

$$\begin{array}{r}
 \$300 \\
 .04 \\
 \hline
 \$12, \text{ int.}
 \end{array}$$

Double the time and take $\frac{1}{2}$ the rate, or double the rate and take $\frac{1}{2}$ the time, as in (a).

(d) Find the interest on \$212.56 for 2 mo. 18 da., at 6%.

$$\begin{array}{r}
 \$212.56 \\
 .013 \\
 \hline
 63768 \\
 2\ 1256 \\
 \$2.76328 = \$2.76, \text{ int.}
 \end{array}$$

Since the interest on \$1 at 6% is $\frac{1}{2}\%$ per month, or $\frac{1}{6}$ of a mill per day, one half the number of months, and one sixth the number of days may be written as the cents and mills of the multiplier.

EXACT INTEREST

238. Exact interest is computed on a basis of 365 days to the year. Exact interest is rarely used except in government calculations. 365 days is 5 days more than common

interest. Five days is $\frac{1}{73}$ of 365, therefore, to find the exact interest, find the interest by any of the above methods and subtract $\frac{1}{73}$ of it.

Find the exact interest :

	PRINCIPAL	RATE	TIME
1.	\$ 700	6 %	80 da.
2.	\$ 800	4 %	75 da.
3.	\$ 975	2 %	15 da.
4.	\$ 812.25	6 %	144 da.
5.	\$ 7368.90	4 %	13 da.

Solve by the most practical method :

6. A savings bank pays 4% interest on its deposits. It lends money at 6%. If its deposits amount to \$237458.50 and its loans amount to \$192546, what is its gain per year?

7. The United States government paid a claim of \$15600 to a creditor, 50 days after it was due. At 5%, how much interest was due the creditor?

8. A corporation is allowed $1\frac{1}{2}$ % annual rate on its average daily balance in a bank, the same to be credited every month. Fill in the allowance for each month :

MONTH	AVERAGE DAILY BALANCE	ALLOWANCE
Jan.	\$ 73210.90	—
Feb.	61740.89	—
March	125608.34	—
April	53217.18	—
May	90020.07	—
June	70663.24	—
July	81210.37	—
Aug.	53721.62	—
Sept.	74889.75	—
Oct.	96271.49	—
Nov.	163618.52	—
Dec.	181241.13	—

COMPOUND INTEREST

239. Compound interest is interest upon principal and added interest. Interest is added at stated periods, 3 months (quarterly), 6 months (semiannually), or 12 months annually).

240. Many transactions are computed by compound interest. Savings banks and insurance companies use this method, and returns on bond investments are reckoned by compound interest.

241. To find compound interest and amount.

Find the compound interest on \$2000 for 3 yr., at 6 %, interest compounded yearly.

SOLUTION 1

\$120, interest for the first year.

\$2000 + \$120 = \$2120, principal for second year.

\$2120 \times 6% = \$127.20, interest for second year.

\$2120 + \$127.20 = \$2247.20, principal for third year.

\$2247.20 \times 6% = \$134.83, interest for third year.

\$120 + \$127.20 + \$134.83 = \$382.03, compound interest.

SOLUTION 2

\$2000 \times 6% = \$120, interest for first year.

\$120 \times 6% = \$7.20, interest on interest.

\$120 + \$7.20 = \$127.20, interest for second year.

\$127.20 + \$120 = \$247.20, compound interest for two years.

\$247.20 \times 6% = \$14.83, interest on interest.

\$120 + \$14.83 = \$134.83, interest for third year.

\$120 + \$127.20 + \$134.83 = \$382.03, compound interest.

NOTE. Solution 1 is generally used.

242. Compound interest is generally computed by using the **compound interest table**.

TABLE SHOWING AMOUNT OF \$1 AT COMPOUND INTEREST FOR ANY NUMBER OF YEARS NOT EXCEEDING TEN

Yr.	2%	3%	3½%	4%	6%
1	1.0200 0000	1.0300 0000	1.0350 0000	1.0400 0000	1.0600 000
2	1.0404 0000	1.0609 0000	1.0712 2500	1.0816 0000	1.1236 000
3	1.0612 0800	1.0927 2700	1.1087 1787	1.1248 6400	1.1910 160
4	1.0824 3216	1.1255 0881	1.1475 2300	1.1698 5856	1.2624 770
5	1.1040 8080	1.1592 7407	1.1876 8631	1.2166 5290	1.3382 256
6	1.1261 6242	1.1940 5230	1.2292 5533	1.2653 1902	1.4185 191
7	1.1486 8567	1.2298 7387	1.2722 7926	1.3159 3178	1.5036 303
8	1.1716 5938	1.2667 7008	1.3168 0904	1.3685 6905	1.5938 481
9	1.1950 9257	1.3047 7318	1.3628 9735	1.4233 1181	1.6894 790
10	1.2189 9442	1.3439 1638	1.4105 9876	1.4802 4428	1.7908 477

NOTE. This is only a small section of a compound interest table. Tables giving all per cents for a long period of time can be secured at any savings bank.

243. To find compound interest by the table :

1. Payable annually, multiply the principal by the amount (shown in the table under the proper rate and time) less \$ 1.

2. Payable semiannually, take $\frac{1}{2}$ the rate and twice the time, less \$ 1.

3. Payable quarterly, take $\frac{1}{4}$ the rate and four times the time, less \$ 1.

NOTE. \$1 is subtracted to find the compound *interest*; the table gives the compound *amount*.

244. If the time should be any time not shown in the table, *multiply* the amounts for the proper aliquot parts of the time; *e.g.*, to compute the compound interest for 20 yr. at 6 %, multiply 1.7908477 by itself, giving 3.2071355.

Find the compound interest on \$400 for 5 yr. at $3\frac{1}{2}\%$.

$$400 \times \$.18768631 = \$75.07$$

See compound interest table, § 242. The amount in the $3\frac{1}{2}\%$ column opposite the 5th year is \$1.18768631. Since this is the *amount*, the compound *interest* on \$1 is \$.18768631. Then the compound interest on \$400 will be 400 times \$.18768631, or \$75.07.

Find the compound interest (by table) :

PRINCIPAL	RATE	TIME
1. \$ 600	3%	8 yr.
2. \$1500	6%	4 yr.
3. \$ 500	4%	3 yr. (semiannually)
4. \$1240	8%	2 yr. (quarterly)
5. \$ 350	4%	3 yr.
6. \$ 590	2%	8 yr.
7. \$ 800	6%	5 yr. (semiannually)
8. \$1000	$3\frac{1}{2}\%$	9 yr.
9. \$2370	3%	4 yr.
10. \$5760	6%	6 yr. (semiannually)
11. \$8320	4%	2 yr.
12. \$5980	2%	7 yr.

SINKING FUNDS

245. A **sinking fund** is a sum set aside each year, at compound interest, sufficient to meet a certain obligation at its maturity. Cities often make public improvements by borrowing money and issuing bonds therefor. The bonds are payable in 5, 10, 15, or more years, as the case may be. To meet these bonds at maturity a sinking fund is established. A series of equal payments is called an **annuity**.

246. Table showing annuity of \$1 at the end of each year from 1 to 10 yr.:

PERIOD	3%	4%	5%	6%
1	1.	1.	1.	1.
2	2.03	2.04	2.05	2.06
3	3.0909	3.1216	3.1525	3.1836
4	4.183627	4.246464	4.310125	4.374616
5	5.309136	5.416323	5.525631	5.637093
6	6.468410	6.632975	6.801913	6.975319
7	7.662462	7.898294	8.142008	8.393838
8	8.892336	9.214226	9.549109	9.897468
9	10.159106	10.582795	11.026564	11.491316
10	11.463879	12.006107	12.577893	13.180795

247. To find an annuity:

1. A certain city makes improvements at an expense of \$300,000 and sells municipal bonds maturing in 10 yr. To pay for them, at 3% interest, what sum must be set aside each year to meet the bonds at maturity?

An annuity of \$1 for 10 yr. at 3% is \$11.463879. Then it will require as many dollars to amount to \$300,000 as \$11.463879 is contained times in \$300,000, or \$26169.16 *Ans.*

2. City bonds to the amount of \$500,000, maturing in 10 yr., were issued. A sinking fund earning 5% was provided. What should be the annual investment in the sinking fund?

3. For the erection of a city hall \$100,000 municipal bonds, payable in 8 yr., are issued. What amount must be laid aside each year at 4% compound interest to redeem the bonds at maturity?

4. A steel company sells \$500,000 worth of 10-yr. bonds, and \$100,000 worth of 10-yr. bonds. The sinking fund earns 4%. What amount must be provided each year to retire both issues at maturity?

BANKS AND BANKING

248. Practically all business is done, directly or indirectly, through banks; they are the money and credit centers of the community. Their most important functions are:

- (1) To receive funds for deposit.
- (2) To make loans.
- (3) To transfer funds. (See Exchange, p. 201.)

By "funds" is meant money, checks, or any other form of credit. The depositor really buys bank credit which he can order transferred as he wishes.

The **credit department**, which handles the loans, is one of the most important departments in a bank. In this department a complete record is kept of the applicant's solvency, viz.:

- (1) An application blank, filled in by the applicant, stating his financial condition.
- (2) The depositor's average balance in the bank, which helps to determine the amount of the loan.
- (3) The applicant's liabilities, both as maker or endorser of notes.

249. Although a bank is organized by individuals, its business is of such a nature that it is subject to very careful regulation by law, both as to its organization and its methods of doing business. There are six kinds of banks, viz.:

(a) **National banks** are chartered by the federal government. They are authorized to issue money (bank notes), and are limited as to the kinds of loans they may make.

(b) **Federal reserve banks** are central banks in the national banking system. They are authorized to issue money (federal reserve notes), and most of their business is transacted with other banks.

(c) **State banks** are organized under state laws; in many details they are like national banks.

(d) **Savings banks** receive money in small amounts, and usually pay interest on deposits.

(e) **Private banks** are companies or individuals doing a banking business under a private name.

(f) **Trust companies** generally have two departments: (1) a banking department doing a general banking business; (2) a trust department which acts as executor of estates, guardian of minors, etc.

250. Banks receive for safe-keeping valuables such as stock certificates, bonds, insurance policies, and the like. Valuables of this kind are usually kept in safety deposit boxes which are rented to individuals.

251. The character of the bank is indicated by its name, as First National, Kent Savings, City Trust and Savings, First State, etc.

252. Deposits. An individual may deposit funds in a bank by (1) opening an account, and (2) taking out a certificate of deposit.

(1) A **checking account** is one in which the funds on deposit are subject to withdrawal by check.

The depositor fills out a **deposit slip**, and the amount deposited is entered by the bank clerk (**teller**) in the bank or **pass book** carried by the depositor. This entry is a record of the transaction for the depositor. The bank, however, opens an account for the depositor in its ledger, and keeps a daily record of deposits and withdrawals. The amount on deposit is called the depositor's **balance**.

DEPOSITED BY

Cary Bros.

ST. LOUIS, MO.,

Mar. 23,

1913

PLEASE LIST EACH CHECK SEPARATELY

		DOLLARS	CENTS
CURRENCY		121	
GOLD			
SILVER		13	20
	CHECKS		
A. C. Co.	31.26		
A. C. Co.	20.15		
M. L. T.	108.26		
F. Hall	7.60		
		167	27
	Total	301	47

A savings account is one in which the funds on deposit draw interest at an agreed rate (usually 3 or 4 %), and are subject to withdrawal only on presentation of the pass book.

Accompanying is a form of pass book used in opening a savings account :

It will be noticed that withdrawals as well as

The Security Savings Bank Co.

N^o 2453 In Acct. with Louis Cooper

Date	Withdrawals	Deposits	Balance
1913			
Jan 1		500	500
" 28		200	
July 1		350	
" 1	By int.		
Dec 11		150	
1914			
Jan 7		600	
" 1	By int.		
" 10		350	
Feb 5		500	

\$100

NUMBER *Void*

THE OHIO NATIONAL BANK

COLUMBUS, OHIO

THIS IS TO CERTIFY
THAT THE SUM OF

\$ 100

HAS BEEN DEPOSITED WITH THIS BANK
PAYABLE TO THE ORDER OF

Void

ON THE DELIVERY AND SURRENDER OF THIS
CERTIFICATE PROPERLY ENDORSED WITH UN-
MATURED COUPONS IF ANY
THIS CERTIFICATE BEARS INTEREST FROM ITS
DATE AT THE RATE OF THREE PERCENTUM PER
ANNUM, PAYABLE QUARTERLY BY THE DE-
LIVERY AND SURRENDER AS THEY BECOME
DUE OF THE COUPONS (PAYABLE TO BEARER)
HERETO ATTACHED
THE PRINCIPAL SUM OF
ONE HUNDRED DOLLARS
IS DUE AND PAYABLE WITHOUT NOTICE, AT THE
MATURITY DATE OF ANY COUPON, OR AT ANY OTHER
TIME WITHOUT INTEREST FROM THE PREVIOUS
COUPON DUE DATE.
COLUMBUS, O.
THE OHIO NATIONAL BANK.
By *Void*
TELLER.

ON 9

THE OHIO NATIONAL BANK
COLUMBUS, OHIO

WILL PAY TO THE BEARER THE SUM OF
SEVENTY-FIVE CENTS
BEING THREE MONTHS INTEREST ON COUPON CERTIFICATE OF DEPOSIT
75¢ No *Void* TELLER.

**CERTIFICATE OF DEPOSIT AND
COUPON**

deposits are entered in the pass book. The depositor's balance can be determined at any time by the entries. This is not true in a checking account.

(2) **Certificates of deposit.** Funds may be deposited in a bank and a certificate issued stating the date, amount, depositor, and person to whom payable.

Certificates of deposit are issued as a convenience to those who wish to get interest on their money and still be able to get it from the bank on demand. They draw a specified rate of interest, commonly from 3 to 4 %, if the money is left in the bank a specified time (usually 3 mo. or more), interest ceasing at the end of 1 yr. unless a new certificate is issued.

The accompanying shows the form of a certificate of deposit on which the interest is payable every 3 mo. by coupon.

253. Withdrawals. Funds are withdrawn from a bank (1) by check, if a checking account; (2) by presentation of the pass book and by signing a receipt, if a savings account; (3) by the surrender of a certificate of deposit.

The following is the form of check commonly used :

No. <u>631</u>	NO. <u>631</u>	FORT WORTH, TEXAS	Dec. 20 1912
To <u>Henry Hawkins</u>		The Citizens National Bank	
For <u>bill of 12/12</u>		PAY TO THE ORDER OF <u>Henry Hawkins</u> \$ <u>38</u> ⁴⁵	
Bal brot ford	DOLLARS	CENTS	<u>Thirty eight</u> ⁴⁵ / ₁₀₀ DOLLARS <u>Samuel Smith</u>
Amt deposited	<u>37</u>	<u>05</u>	
Total			
Amt this Check	<u>38</u>	<u>45</u>	
Bal car'd ford	<u>33</u>	<u>16</u>	

The "stub" of the check is for the convenience of the depositor in keeping a record of his transactions with the bank. When would the balance, as shown on the stub, differ from that in the bank's ledger?

254. Unless withdrawn, interest is allowed on savings deposits twice a year, on Jan. 1 and on July 1; thus compound interest is paid. No interest is paid on parts of a dollar, nor usually on any money not left during the entire interest period.

Savings banks may demand 30 days' notice of the withdrawal of funds.

1. In the savings account of Louis Cooper, p. 159, enter the proper interest due him at 3% on July 1, 1913, and on Jan. 1, 1914, and compute his balance on Feb. 6, 1914.

2. The City Railway and Light Co.'s balance in the Fourth National Bank on Dec. 9 is \$34,786.49. The company makes a deposit of \$18,726.12, and draws a check for its weekly pay roll for \$20,112.47. What is the company's balance at the opening of the bank on Dec. 10?

3. A coupon certificate of deposit for \$400, dated Feb. 23, 1912, bears interest at 3% payable quarterly after date. What is its value on Aug. 23, 1912?

255. An indorsement is anything written on the back of a commercial paper, which refers to the paper itself.

256. Checks, certificates of deposit, and other forms of commercial paper may be transferred from one person to another by indorsement, unless drawn to the contrary; they are called **negotiable paper**. The omission of the words “or order,” or “to the order of,” makes paper **nonnegotiable**.

Such indorsements may read: “Pay to the order of —,” “Pay to — or order,” “Pay to bearer,” etc. The following indorsements, supposed to be written on the back of the check on p. 161, show (on the left) the indorsement used in cashing the check; (on the right) the indorsement used in transferring the money to Charles Elwert.

HENRY HAWKINS	<i>Pay to CHARLES ELWERT or order</i> HENRY HAWKINS
---------------	--

257. Postal savings banks have been established by the United States government at certain post offices.

Postal savings certificates are issued in denominations of \$1, \$2, \$5, \$10, \$20, and \$50, each bearing the name of the depositor and other necessary information. Deposits up to \$100 monthly are allowed to any one person.

Interest at 2% annually is paid on each certificate left on deposit for a full year. Compound interest is not allowed.

United States bonds, bearing interest at $2\frac{1}{2}\%$, to any amount not exceeding \$500 to one person, will be issued to a depositor upon surrender of certificates.

258. Loans and discounts. Banks lend money on **notes**, the payment of which is secured by some valuable assigned as **security** or **collateral**.

Loans are made on “collateral notes,” on “judgment notes,” and the like. Forms of notes differ for different kinds of business.

259. Notes may be secured in the following ways :

1. By the indorsement of one or more persons who thus become liable for payment.
2. By the deposit of collateral.
3. By the mortgaging of property.

Grand Rapids, Mich. <u>Nov. 24</u> 191 <u>2</u> No. _____	
Sixty days after date (without grace) for value received I promise to pay	
to the order of the City Trust & Savings Bank \$760 ⁰⁰	
Seven hundred sixty _____ Dollars	
with interest at the rate of seven percent per annum from maturity until paid	
PROTEST WAIVED	Payable at <u>L. G. Hurst.</u>
The City Trust & Savings Bank	} Address <u>221 Astor Bldg.</u>
Due _____ Grand Rapids, Mich.	

FORM OF NOTE

260. Loans may be classified as follows :

(a) **Investment loans**, made upon collateral security which can readily be turned into cash.

(b) **Industrial loans**, made to manufacturers, merchants, and farmers for discounting bills, extending trade, moving crops, etc.

(c) **Capital loans**, made to manufacturers and merchants who want permanent capital for starting a business and expect to repay the loans from the earnings of the business.

(d) **Mortgage loans**, made usually on real estate as security.

261. Most banks make **short-time loans**, the notes usually falling due in 30, 60, or 90 days; or **demand loans** which may be called in (payment demanded) at any time.

Long-time loans are made by savings banks and by trust companies. Most of the money lent on mortgage security is lent by individuals or by insurance companies.

262. Organizations known as Building and Loan Associations make loans on real estate, especially for building purposes. In order to secure such a loan one must become a stockholder in the association, the payments for stock and the repayment of the loan being made in weekly or monthly installments.

263. The date on which a note or obligation becomes due and payable is called the **date of maturity**.

BANK DISCOUNT

264. **Bank discount** is an amount deducted from the face of a note as a consideration for the use of the money before maturity.

If the note bears interest, the discount is deducted from the face of the note plus the interest.

265. The **proceeds** is the amount left after the discount has been deducted.

266. **Time of discount** is the actual time from the date the note is discounted to the day of maturity.

267. Interest is figured by calendar years, months, or days, according to the wording of the note or draft. Bank discount is reckoned on actual time for parts of a year, not counting the day of discount.

268. *Three days of grace* beyond the date of maturity on notes and drafts were formerly allowed in most states. However nearly all states have discarded this practice and now have the date of payment the same as the date of maturity.

NOTE. Problems in bank discount are based upon percentage and common interest. No days of grace are considered in this text.

EXERCISES

1. Find the bank discount and proceeds of the following note, which was discounted Jan. 6, 1912, at 8%.

Denver Colorado	Nov. 1	1911	No. 58
Four months after date (without grace) for value received. I promise to pay			
to the order of James T. Fox		\$5680 ⁴⁰ / ₁₀₀	
Fifty six hundred eighty ⁴⁰ / ₁₀₀ Dollars			
with interest at the rate of six per cent. per annum from maturity until paid.			
PROTEST WAIVED Payable at			
The City Trust & Savings Bank		Newton Speckman	
Due Denver Colorado		Address	

SOLUTION

Nov. 1, 1911 + 4 mo. = March 1, 1912, date of maturity.

TIME OF DISCOUNT

31
6
25 da. in Jan.
29 da. in Feb.
1 da. in March.
55 da.

INTEREST

\$5680.40, principal
56.804, interest 60 da. } 4 mo.
56.804, interest 60 da.
\$5794.01, amount to be discounted

\$57.94, discount for 60 da. at 6%
4.828, discount for 5 da. at 6%
53.112, discount for 55 da. at 6%
17.704, discount for 55 da. at 2%
70.82, discount for 55 da. at 8%

\$5794.01 - \$70.82 = \$5723.19, proceeds.

Find the discount and proceeds of:

2. \$1500 with interest at 4% for 6 mo. from April 10, discounted June 8 at 6%.

3. \$875 with interest at 6% for 90 da. from Jan. 3, discounted Feb. 5 at 7%.

4. \$3100 without interest for 1 yr. from July 7, 1914, discounted Dec. 10, 1914, at 6 %.

5. \$4750 with interest at $4\frac{1}{2}$ % for 120 da. from June 11, discounted Aug. 1 at 7 %.

269. Bankers make use of a table for finding the number of days between dates.

TIME TABLE

FROM	To											
	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

EXPLANATION OF TABLE. From Jan. 10 to Jan. 10 is 365 da.; from Jan. 10, 1912, to Jan. 24, 1913, is 365 da. + 14 da.; from June 6 to Nov. 30 is 177 da. (24 + 153).

NOTE. When banks discount a paper, they often charge for their services a certain per cent, ranging from $\frac{1}{10}$ % to $\frac{1}{2}$ %, on the face of the paper, or the face plus the interest; there is usually a minimum charge of 10¢. This charge is called **collection**. The bank discount plus the collection charge is the total charge.

EXERCISES

Find the bank discount and proceeds on the following. Use the table (§ 269) for finding the number of days.

	FACE	DATE	DATE DUE	RATE	DATE DISCOUNTED	RATE DISCOUNTED
1.	\$ 700	1/ 2/13	7/ 2/13	5%	3/ 4/13	6 %
2.	1460	9/ 3/11	11/ 3/11	6	10/21/12	5
3.	1500	7/ 5/12	1/ 5/13	7	10/12/12	6
4.	50	10/31/12	11/30/12	6	11/ 5/12	5½
5.	3146.75	1/24/13	5/24/13	5	3/20/13	8
6.	8154.09	2/27/13	10/27/13	4	6/12/13	6
7.	321.44	3/15/13	4/15/13	5	4/10/13	7
8.	750	5/22/13	8/22/13	4½	7/31/13	5½
9.	1000	1/2/12	1/ 2/14	6	11/24/13	6
10.	150	2/ 3/12	2/ 3/13	8	1/ 6/13	7

11. The following note was discounted Jan. 1, 1913, at 6 %; collection, $\frac{1}{4}$ %, computed on the face of the note. Find the bank discount and proceeds after allowing for collection.

\$3259

NASHVILLE, TENN., Dec. 1, 1912.

Sixty days after date, for value received, we promise to pay Wm. A. Pierson or order,

Thirty-two hundred fifty-nine no/100 dollars, at the Traders' National Bank, in Nashville, Tenn., with interest at the rate of eight per cent per annum after maturity until paid.

CARROLL BROS. AND CO.

12. Find the proceeds on a 3 mo. note for \$175, interest at 8 % from Oct. 5, discounted Oct. 16 at 10 %; collection, $\frac{1}{8}$ %.

13. A 90-da. draft for \$350 was discounted Jan. 3, 1913, the day of acceptance, at 6 %; collection, $\frac{1}{8}$ %. Find the proceeds.

PRESENT WORTH AND TRUE DISCOUNT

270. The **present worth** of a debt is the sum which, put at interest, will produce the value of the debt at maturity.

271. The **true discount** is the difference between the amount due at maturity and the present worth.

NOTE. Bank discount is rapidly replacing present worth and true discount.

1. Find the present worth and true discount of a debt of \$500, due in 6 mo., if the rate of discount is 6%.

\$1, with int. at 6% for 6 mo., will amount to \$1.03.

If \$1 amounts to \$1.03 in 6 mo. at 6%, it will take as many dollars to amount to \$500 in that time as \$1.03 is contained in \$500, or 485.44.

Then \$485.44 is the present worth.

\$500 - \$485.44 is the true discount, \$14.56.

Find the present worth and true discount of:

2. \$1700 for 3 mo. at 4%

3. \$2500 for 4 mo. at 5%

4. \$5000 for 10 mo. at 6%

5. \$900 for 2 yr. and 6 mo. at 3%

6. What is the difference between true discount and bank discount on a note for \$1000 for 3 mo. at 8%?

7. Find the simple interest, true discount, and bank discount on a note for \$3000 for 6 mo. at 6%.

8. A dealer's price on an automobile is \$1200, 4 mo. time, or \$1150 cash. At what per cent does he figure money?

9. A bill of goods is invoiced at \$2500 on 3 mo. credit. For how much must a note be drawn with interest at 6% to cancel the debt?

10. What note at 6% interest must be given March 1 to cancel the following bill at maturity?

NEW YORK, N.Y., 2/18/12.

C. H. GARRISON

DETROIT, MICH.

Salesman C. H.

Order taken 8/30.

In account with

2/10, net 60

March 1 dating

PHILLIPS-JONES COMPANY

O U 1	3	doz.	@	4.50	each
O U 3	1 6/12	"	@	4.50	"
898	4 6/12	"	@	4.50	"
898 B	1 6/12	"	@	4.25	"
B R H 91	1 3/12	"	@	8.50	"
S E 1	1 9/12	"	@	8.50	"
B V C 61	1 3/12	"	@	8.50	"
					Total

PARTIAL PAYMENTS

272. Partial payment is part payment on a note, draft, or other form of obligation.

When a payment is made, an indorsement is entered on the back of the paper, somewhat after the following form :

Rec'd on the within note

Jan. 1, 1914, \$50

March 2, 1914, \$75

NOTE. The ordinary practice, especially with banks, is to take separate notes for whatever payments are agreed upon at the time the loan is made. Thus, if a borrower wishes to borrow \$150 for 3 mo., and repay \$50 a month, the bank would probably take 3 notes for \$50 each, one due in 30 da., one in 60 da., and one in 90 da.

273. The two methods in general use in computing partial payments are the United States Rule and the Merchants' Rule.

274. The **United States Rule** is authorized by the United States Supreme Court, and is used when the partial payments are made on interest-bearing notes having 1 yr. or more to run.

- (a) Interest must not be paid on interest.
- (b) Accrued interest must be paid before the payment can reduce the amount of the debt.

1. What was the balance due on a note for \$2000 dated June 10, 1911, to run 1 yr. with interest at 6%, the following payments having been made: Aug. 3, 1911, \$500; Dec. 15, 1911, \$25; Feb. 3, 1912, \$500? (Compound time is always used.) (Use the 6 % method.)

SOLUTION

Face of note	\$ 2000.
Int. from June 10, 1911, to Aug. 3, 1911	17.67
Amount due Aug. 3, 1911.	2017.67
Payment Aug. 3, 1911	500.
Balance due Aug. 3, 1911	1517.67
Int. from Aug. 3, 1911, to Dec. 15, 1911	\$33.39
As this amount (\$33.39) exceeded the amount of the payment, int. and payment must be carried over to the next payment	
Int. from Dec. 15, 1911, to Feb. 3, 1912	12.14
Total int. due	45.53
Amount due Feb. 3, 1912	1563.20
Total payment (\$25 + \$500)	525.
Balance due Feb. 3, 1912	1038.20
Int. from Feb. 3, 1912, to June 10, 1912	21.98
Amount due at settlement	\$1060.18

Find the balance due on each of the following notes :

	DATE	FACE	TIME TO RUN	RATE	PAYMENTS
2.	Sept. 1, 1909	\$ 10000	4 yr.	6 %	March 1, 1910, \$ 300 Dec. 1, 1910, \$ 50 June 1, 1911, \$ 400
3.	Feb. 5, 1900	\$ 2000	2 yr.	4 %	May 1, 1900, \$ 200 Aug. 1, 1901, \$ 10

275. The **Merchants' Rule**, while not a strict rule of law, is generally used when the interest-bearing note runs 1 yr. or less.

(a) The note bears interest until date of settlement.

(b) Each payment bears interest from its date until date of settlement.

NOTE. Some business men use exact time, while others use compound subtraction.

1. Face of note, \$1000; time, 9 mo. from Jan. 1; rate, 6 %. Indorsements: Feb. 1, \$100; June 1, \$200. What amount was due at maturity? (Compound time.)

CONDENSED FORM OF SOLUTION

DATE	FACE	TIME	RATE	INTEREST	AMOUNT
Jan. 1	\$1000	9 mo.	6 %	\$45.00	\$1045
Feb. 1	100	8 mo.	6 %	4.00	\$104
June 1	200	4 mo.	6 %	4.00	204
					Balance due 737
					\$1045 \$1045

Find the balance due on each of the following, using compound subtraction to find the time in the 2d and 3d, and exact time in the 4th and 5th.

	DATE	FACE	TIME	RATE	INDORSEMENTS
2.	Feb. 1, 1912	\$4000	10 mo.	6 %	April 3, 1912, \$500 June 1, 1912, 100 Oct. 10, 1912, 200
3.	Jan. 25, 1910	1500	7 mo.	7 %	Feb. 18, 1910, 150 July 2, 1910, 350
4.	Aug. 9, 1908	850	5 mo.	6 %	Sept. 28, 1908, 75 Nov. 5, 1908, 50 Jan. 1, 1909, 100
5.	Dec. 10, 1903	2000	9 mo.	5 %	Jan. 20, 1904, 200 Feb. 25, 1904, 300

MISCELLANEOUS PROBLEMS

GROUP 1

1. From the equation $PRT = I$, find R .
2. By applying the formula, find I in the following :

P	R	T
\$260.	6 %	1 yr.
720.	4 %	6 mo.
8400.	5 %	3 mo.
568.72	7 %	90 da.
85.60	8 %	4 mo.

3. By the cancellation method find the interest on the following :

\$98.60 for 72 da. at 6 %
 \$321.05 for 90 da. at 5 %
 \$426.17 for 35 da. at 7 %
 \$1260.09 for 63 da. at 6 %
 \$815. for 3 mo. at 8 %

4. By the 60-da. method find the interest on the following :

\$621.50 for 4 mo. at 6 %
 \$283.17 for 5 mo. at 5 %
 \$12620.70 for $\frac{1}{2}$ yr. at $4\frac{1}{2}$ %
 \$84.62 for 75 da. at 3 %
 \$161.13 for 112 da. at $3\frac{1}{2}$ %

5. By the 6 % method find the interest on the following :

\$378.50 from Feb. 2, 1913, to April 10, 1913, at 6 %
 \$216.06 from 11/3/12 to 2/1/13 at 8 %
 \$7615.83 from Oct. 23, 1912, to Jan. 13, 1913, at 5 %
 \$70 from 1/3/13 to 4/19/13 at 7 %
 \$169.69 from 1/15/13 to 2/28/13 at 4 %

6. By the interest table, p. 149, find the interest on the following :

\$1000 for 8 mo. 3 da. at $4\frac{1}{2}\%$

\$300 for 3 mo. at 6%

\$279 for 1 mo. 15 da. at 5%

\$1628 for 1 yr. 2 mo. 11 da. at $5\frac{1}{2}\%$

\$65 for 36 da. at 7%

7. Find the exact interest (p. 151) on :

\$1427 for 48 da. at 6%

\$216.60 for 37 da. at $6\frac{1}{2}\%$

\$98.54 for 3 mo. 12 da. at 5%

\$627.43 for 1 mo. 15 da. at $4\frac{1}{2}\%$

\$13261.82 for 1 yr. 2 mo. 27 da. at 7%

8. Find, by the most convenient method for each, the interest on the following :

\$3726.81 from April 5, 1911, to June 3, 1913, at 6%

\$281.54 from Oct. 13, 1912, to Jan. 5, 1913, at 7%

\$2200 from July 3, 1910, to Aug. 6, 1914, at 8%

\$75 for 7 mo. 8 da. at $2\frac{1}{2}\%$

\$125 for 9 mo. 22 da. at 4%

9. A man lent £ 2159 5s. 6d. at 6% . Find the interest yearly.

10. How much money must I invest at 5% in order to receive a semiannual income of \$1268.50?

GROUP 2

1. A man builds a house costing \$3200. He pays \$1400 cash and gives three equal notes for the balance due, payable in 1, 2, and 3 yr. respectively, with interest at 5% . If he pays all interest due at the end of each year, what amount does he pay at the end of each year in paying off the indebtedness?

2. A merchant bought a bill of goods on the following terms: 3% cash, net 90 da. At what rate can he afford to borrow money in order to pay cash?

3. A bank pays 3% interest on deposits amounting to \$628,453.27; its loans average \$1,462,817.61 at 5%, and \$1,316,478.55 at 6%. What is the bank's yearly profit?

4. A city's bonded debt is \$1,031,000. At 4%, what interest does it pay annually?

5. A merchant bought \$5000 worth of goods, terms net 60 da. At the end of the 60 da. he had sold 75% of the goods at a profit of 20% on the cost, and the rest of them at cost. How much did he gain?

6. A gave his 60-da. note to a bank for \$200 at 6% interest. If the bank deducts the interest in advance, how much does he receive for the note?

7. On April 1, 1910, A bought a bill of goods amounting to \$1500. He gave his note for the amount, interest at 5%. How much did he pay at settlement, Nov. 15, 1910?

8. A company bought 250 bales of cotton at \$47.64 per bale, giving a 30-da. $5\frac{1}{2}$ % note in payment. How much interest did it pay at maturity?

9. A farmer bought an 80-acre farm for \$50 per acre. He paid \$800 down, and gave four equal notes for the balance due, payable in 1, 2, 3, and 4 yr. respectively, interest at 6%, payable annually. How much did he pay at the end of each year until all the notes were paid?

10. A railroad company buys 1000 gondola cars at \$1000 each, giving in payment 20 series of 50 notes to each series, all dated July 1, 1913. One series matures at the end of each 6 months, and all bear interest at 6% per annum from date, interest payable semiannually. What amount must the company pay at the end of the first 6 months? at the end of the second 6 months?

GROUP 3

1. A city authorizes the issuance of \$350,000 10-yr. bonds at 4% for the purpose of building a viaduct. What amount must be paid to a sinking fund earning 3% in order to redeem the bonds at maturity? (Use table, page 156.)

2. A depositor has \$1000 in the savings bank at 3% interest. He draws out the money and buys a lot which he sells 6 mo. later for \$1100. If he pays \$15 taxes on the lot, and \$20 selling commission, what per cent does he make on his \$1000 in excess of what it would have earned for him if left in the bank?

3. An electric light company borrows, on a $5\frac{1}{2}\%$ bond issue, \$100,000 for extensions and improvements. What interest does it pay annually on the debt, and what amount must it deposit with the trustee of the sinking fund earning 4% to redeem the bonds in 5 yr.?

4. What is the difference between a checking and a savings account at a bank?

5. The bank clearings in New Orleans on a certain day were \$1,841,109, as against \$1,524,060 for the corresponding day the year before. What was the percentage of increase?

6. A coupon certificate of deposit for \$600, drawing interest at 3%, is dated Oct. 28, 1912. If all the coupons are left attached, what is the value of the certificate on Aug. 3, 1913?

7. A cotton dealer sold 175 bales of cotton at \$48.32 per bale, and accepted in payment a 90-da. note bearing interest at 5%. If he discounts the note at his bank at 5%, how much does he receive for it?

8. A bank discounts a 60-da. draft at 6%, allowing 5 da. for collection and return. If the face of the draft is \$675.84, what are the proceeds, there being no charge for

collection? (Time for collection and return must be added to time of draft.)

9. A jobber buys goods from a manufacturer to the amount of \$27,600, and gives his note in payment for 4 mo. at 5%. The jobber sells the goods to customers at a profit of $12\frac{1}{2}\%$ on the cost, taking 60-da. 6% notes in payment. At the end of 60 da., receiving the money on his customer's notes, he discounts his note to the manufacturer at 6%.
- What is his profit on the transaction?

10. A, holding B's 7% 60-da. note for \$1200, has it discounted at the bank for the full time at 6%. At the end of 30 da., having idle funds, A buys the note back from the bank, discounting it at 6%. What net amount does A receive in interest?

GROUP 4

1. A retail merchant sold out a stock of goods and invested 70% of the money in a farm which paid him a yearly rental of \$600. If the rate of income on his farm investment is $8\frac{1}{3}\%$, for what amount did he close out his stock?

2. The profits of a manufacturing business the first year are 40% of the capital employed; 75% of this gain is reinvested in the business for the 2d year. The 2d year shows a profit of 45% on the capital employed, and all this profit is allowed to remain in the business for the 3d year. What per cent of the original capital does the business start with for the 3d year?

3. What rate of commission is charged for selling goods for \$562.25, the net proceeds of the sale being \$541.17?

4. A bought a bill of goods to the amount of \$640, terms 3%/10, net 30. In order to secure the 10-da. discount he discounted his 60-da. note at the bank at 6%. Find the face of the note.

5. Find the difference between the simple interest and true discount on \$2500 for 90 da. at 5%. Find the difference between simple interest and bank discount.

6. If the John F. Price Co. accepted Frank Mueller's 60-da. note in payment of the following bill, for what amount could it be discounted at the bank, interest being at the rate of 6%?

Sept. 20, 1914.

FRANK MUELLER
OGDEN, UTAH.

In account with
THE JOHN F. PRICE CO.

1	suit	\$25	00
2½	doz. hdkfs. @ 2 for 25¢		
6	pr. sacks @ 3 pr. for \$1		
5	shirts @ \$1.75 each		
6	ties @ \$4 per dozen		

7. What is the difference between the true and bank discount on \$3000 for 150 da. at 6%?

8. Find the balance due at maturity by United States Rule on the following note :

Face of note, \$3500; date, Jan. 10, 1913; time to run, 2 yr. 6 mo.; rate of interest, 6%. Indorsements are as follows : 9/25/13, \$400; 11/1/13, \$500; 2/8/1914, \$200; 6/15/14 \$1000.

9. On his son's 11th birthday, a man deposited in a savings bank a sum sufficient to amount to \$3000 on the son's 21st birthday. If the bank credits 4% interest annually, how much was deposited? (§ 242.)

10. Find the interest on £ 200 10s. 5d. for 2 mo. 15 da., at 6%.

GROUP 5

1. A contractor agrees to build a bridge for \$15000. He employs 15 men for 120 da. at \$2 per day each, 1 engineer for 108 da. at \$5 per day, 2 teams for 123 da. at \$4.50 per day each. Materials are billed to him at a discount of $2\frac{1}{2}\%$, the bills calling for \$7186.20. In order to pay his men promptly, and to discount his bills, the contractor borrows money from the bank at 6% as follows: \$2500 for 1 mo., \$2500 for 2 mo., and \$3000 for 3 mo. His surety bond for the safe completion of his contract costs him \$225. What is the profit on the contract?

2. A man borrowed \$50 and agreed to pay back \$10 a month for 6 mo. Ignoring interest on the monthly payments, at what rate does he pay interest on the loan?

3. A man invested \$5000 on Jan. 15, buying stocks which he sold later for \$5300. He reinvested this amount, and sold out on May 24 at an advance of 2% on his investment. What rate of income has his money earned?

4. A jobber gives a customer a discount of 10% from list prices, with an additional discount of 2% for cash. What was the amount of the bill at list prices if the customer paid \$2364.85 cash?

5. A bill for \$2534.20 was due a manufacturer Jan. 2. He carried the account until May 15 without interest. If money is worth 5%, this was equivalent to what discount?

6. A farmer owns a farm worth \$15000 with its stock and implements. His accounts for a year are as follows:

DR.	CR.
\$1000, labor, self and wife	\$1400, sale of wheat
420, living expenses	280, sale of stock
112, extra help	150, sale of corn
<u>150, taxes and incidentals</u>	<u>321, sale of other products</u>

What rate of income does the farm pay above expenses?

7. A man gave his note Feb. 15, 1909, for \$1250.75, at 7%. Some time afterward he canceled the note by paying \$1354.56 in full. What was the date of cancellation?

8. On March 29, 1908, J. R. Anderson, of Fort Worth, Tex., bought of Scott Freeman 42 head of cattle at \$42.50 a head, and gave in payment an 8% 90-da. note payable at the Citizens' National Bank of Fort Worth, J. F. Baker becoming his security. Write the note.

9. Scott Freeman discounted the above note at the bank on April 7 at 6%. What were the proceeds?

10. A city contracts for paving as follows:

3000' brick paving on A St., 36' wide, at \$1.68 per square yard.

1600' asphalt paving on B St., 30' wide, at \$1.80 per square yard.

7500' macadam paving on C St., 24' wide, and 8'' deep, at \$2 per cubic yard.

13800' curbing on C St., at 51¢ per foot.

To pay for the paving the city issues \$50,000 5-yr. 4% bonds. Any balance left out of the \$50,000 after paying for the paving is to be deposited in a bank at $2\frac{1}{2}\%$, interest to be paid semiannually and to be applied to the bond charges each six months. What amount must the city appropriate to meet the interest and sinking fund needs each year if the sinking fund earns 3%?

TAXES

276. Taxes are sums of money charged against persons or property for public purposes.

277. Taxes are of two kinds, direct and indirect.

278. Direct taxes are sums levied upon persons (income tax), property (property tax), or business (license fee).

The nature of some direct taxes is indicated by the name, such as. "Inheritance," "Corporation," "Income Tax," etc.

279. Indirect taxes are duties levied upon imported goods (called tariffs), or licenses charged on the manufacture of liquor or tobacco products (called excise or internal revenue).

280. Taxes are generally assessed and made payable in money, except road taxes which are often made payable in day's work.

The value at which property is assessed is determined by officials called assessors.

281. In solving tax problems use the principles of percentage.

The assessed valuation is the base.

The tax rate is the rate.

The tax is the percentage.

282. Formulas :

Assessed valuation \times rate = tax, or $BR = T$.

Tax \div rate = assessed valuation, or $\frac{T}{R} = B$.

Tax \div assessed valuation = rate, or $\frac{T}{B} = R$.

283. The tax rate may be expressed in two ways; by a per cent, as $1\frac{3}{10}\%$, or by so much on \$100, as \$1.30 per \$100.

Find the tax on property valued at \$2000 at a rate of $1\frac{1}{5}\%$.

If $BR = T$,

then $\$2000 \times .012 = \24 , tax.

Fill in the missing parts:

	VALUATION	TAX RATE	TAX
1.	\$ 5500	$1\frac{1}{3}\%$	_____
2.	\$ 1500	.0125 %	_____
3.	\$ 12000	.014 %	_____
4.	\$ 6250	_____	\$ 93.75
5.	_____	.02 %	\$ 112.62
6.	_____	$1\frac{7}{8}\%$	\$ 187.50
7.	\$36000	_____	\$ 420.
8.	_____	.022 %	\$ 56.32

APPORTIONMENT OF TAXES

284. Direct taxes are apportioned in the following way:

Suppose the property value in a state is \$2,000,000,000; the valuation of X county is \$25,000,000, and of the city of Y \$7,000,000.

Suppose also that the amount of money to be raised, exclusive of licenses, permits, etc., is \$14,000,000 for state purposes. X county must pay its proportionate part of this \$14,000,000, or such a part of it as \$25,000,000 is of \$2,000,000,000.

$$\frac{\$25000000}{\$2000000000} = \frac{25}{2000} = 1\frac{1}{4}\% \text{ of the whole amount.}$$

$$\$14,000,000 \times .01\frac{1}{4} = \$175000, \text{ amt. to be raised by X county.}$$

Further, if \$25,000 is needed for county purposes, then X county must raise a total of \$200,000.

Similarly, the city of Y, with a valuation of \$7,000,000, must raise $\frac{7000000}{25000000}$ of the amount to be raised by the county, or $\frac{7}{25}$ of \$200,000 = \$56000, Y's share of the county tax.

If the city of Y must raise \$44000 for city expenses, its total tax will be \$56000 + \$44000, or \$100,000 for all purposes.

To find the rate of taxation, divide \$100,000, the amount to be raised, by \$7,000,000, the total valuation of the town's property.

$$\frac{\$100000}{\$7000000} = 1\frac{3}{7} \%, \text{ the rate.}$$

1. A property has an assessed valuation of \$10,000. What is the tax at the rate of \$.001 on a dollar?

2. If the assessed valuation is \$5700 and the tax rate $1\frac{2}{3}\%$, what is the tax?

3. What is the tax on \$5000 at the rate of \$.80 per \$100?

4. A corporation paid taxes as follows: a corporation tax (fee) of \$5, $1\frac{7}{8}\%$ on real estate assessed at \$37,250, and an income tax of \$37.50. What was its total tax?

5. A man paid \$230 taxes on real estate at $1\frac{3}{8}\%$. At what amount was his property assessed?

6. An owner paid \$201.31 taxes on property assessed at \$8750. What was the rate of taxation?

7. An estate pays to the state \$7240.60 as an inheritance tax at 1.5%. On what amount is the tax levied?

8. The total property valuation of a county is \$33,264,780, and the amount to be raised by taxation is \$650,000. What is the tax on property valued at \$4500?

9. A farmer owns property, real and personal, assessed at \$5235. His poll tax is \$1, property tax \$75, and he is assessed \$4 road tax. What per cent of his property valuation must he set aside for taxes?

INDIRECT TAXES

285. Customs or duties are levied by the government for two reasons :

1. For protection of American industries.
2. For raising revenue.

286. Customs or duties are of two kinds,—specific and ad valorem.

287. A **specific duty** is a fixed sum of money levied upon each article regardless of its value.

288. An **ad valorem duty** is a certain per cent levied upon the market value of the article in the country from which it is imported.

289. Allowances for the weight of boxes, etc. (called tare), and for leakage and breakage, are made before estimating duties.

The long ton of 2240 lb. is used in reckoning import duties.

290. A **tariff** is a list of dutiable articles with their legal rate.

291. A **free list** is a list of articles on which no duty is charged.

292. The following table is taken from the Tariff Act of 1913.

IMPORT DUTIES

ARTICLE	SPECIFIC DUTY	AD VALOREM DUTY
Bicycles		25 %
Butter	2½¢ per pound	
Cotton cloth, unbleached		7½ %
Cotton clothing, ready-made		30 %
Leather “glacé” gloves, ladies’ 14 in. long or less	\$1 per dozen	
ladies’ 17 in. long	\$1.75 per dozen	
Oilcloth		25 %
Woolen clothing, ready-made		35 %

293. To find a specific duty.

What is the duty on 175 doz. ladies' gloves, 14 in. long?

$$175 \times \$1 = \$175, \text{ specific duty.}$$

Using the rates given above, find the duty on:

1. 200 doz. ladies' leather gloves, 12 in. long.
2. 500 doz. ladies' leather gloves, 17 in. long.
3. 1500 lb. of butter.

294. To find an ad valorem duty.

What is the ad valorem duty on imported merchandise, valued at £ 300 10s., if the duty is 25 %?

$$£ 300 \text{ } 10s = £ 300.5.$$

$$\$4.8665 \times 300.5 = \$1462.38, \text{ value in United States money.}$$

$$\frac{1}{4} \text{ of } \$1462 = \$365.50 \text{ ad valorem duty.}$$

NOTE. The duty is reckoned on the nearest dollar.

Find the duty on:

1. 300 yd. of oilcloth, invoiced at £ 6 8s. 10d.
2. Woolen clothing invoiced at £ 510.
3. A merchant imported merchandise valued at £8000. The duty was 50 %. What was the total cost in U. S. money?
4. Find the total cost to the importer of the following shipment of gloves, the import duty on ladies' gloves being \$1 per dozen, and on the men's gloves, \$2.50 per dozen, the freight being \$18.20.

NO. ARTICLE	PRICE	AMT.		DUTY		TOTAL	
140 pr. 14'' ladies'	\$ 1.45						
170 pr. men's	1.10						
	Total						
	Freight						
	Total cost						

INSURANCE

295. **Insurance** is an agreement by one party, for a consideration, to indemnify (pay) another party in case of loss.

The loss may be from any cause stipulated in the agreement, and the different kinds of insurance take their names from the various causes. Fire insurance is insurance against loss by fire; life insurance is insurance against loss of life; accident, against injury from accident; etc. Other kinds of insurance are marine, burglary, plate glass, automobile, hail, liability, etc.

296. The agreement between the two parties is called the **policy**. The amount paid for insurance is the **premium**.

297. Fire and life insurance are the most important; and a consideration of these two kinds will furnish the general basis for calculating any kind of insurance. Hence, only these two will be treated in this book.

FIRE INSURANCE

298. **Fire insurance** covers loss or damage to property by fire, water, smoke, and chemical extinguishers.

299. Two kinds of insurance are in general use: valued policies and open policies.

300. A **valued policy** states the amount to be paid in case of loss.

301. An **open policy** leaves the amount to be paid in case of loss open to evidence. This form of policy is generally used.

302. The value of the policy, or the amount of risk assumed, is the base.

The rate of premium is the rate.

The premium is the percentage.

303. The rate of premium is sometimes stated as a per cent of the amount insured, and sometimes as a certain rate on each \$100 of insurance.

304. To find the premium.

A house is insured for \$3000 at $1\frac{1}{2}\%$ premium. Find the cost of insurance.

$$1\frac{1}{2}\% \text{ of } \$3000 = \$45, \text{ premium.}$$

$$(B \times R = P.)$$

Find the premium on each of the following policies:

	FACE OF POLICY	RATE OF PREMIUM
1.	\$ 3800	$\frac{1}{2}\%$
2.	\$ 5600	$\frac{1}{3}\%$
3.	\$48200	$\frac{2}{5}\%$
4.	\$12000	30 ¢ per \$100
5.	\$ 7400	$2\frac{1}{4}\%$
6.	\$60500	42 ¢ per \$100
7.	\$18300	21 ¢ per \$100
8.	\$51200	1 %
9.	\$17100	$3\frac{1}{2}\%$
10.	\$25500	60 ¢ per \$100

305. The insurable value of buildings is roughly estimated by the following method:

Dwellings from 5 ¢ up per cubic foot.

Barns from 2 ¢ to 3 ¢ per cubic foot.

Deduct one third for depreciation and owner's risk.

$$30 \times 25 \times 28 = 21000 \text{ cubic feet}$$

	8¢	
	<u>\$ 1680</u>	
Less $\frac{1}{3}$	560	
	<u>\$ 1120,</u>	insurable value.

$$\frac{8000}{12000} = \frac{2}{3}$$

$\frac{2}{3}$ of \$7500 = \$5000, amount paid.

What amount would be paid under an ordinary policy?

310. A stock of merchandise is insured as follows :

In Company A for \$3000.

In Company B for \$2000.

In Company C for \$1000.

What would each company pay in the event of a loss of \$3000?

\$ 3000

2000

1000

\$ 6000, total insurance

$\frac{3}{6}$ of \$3000 = \$1500, amount Company A pays.

$\frac{2}{6}$ of \$3000 = \$1000, amount Company B pays.

$\frac{1}{6}$ of \$3000 = \$ 500, amount Company C pays.

EXERCISES

1. A dwelling $30' \times 60' \times 25'$, costing 10¢ per cubic foot, is insured for 5 yr. at a rate of \$1.35 per \$100. Find the insurable value and the premium.

Note that the rate is for five years, not for one year.

2. A barn $40' \times 100' \times 40'$, costing 3¢ per cubic foot, is insured for 3 yr. at a rate of \$1 per \$100. What is the insurance company's net loss if the barn is totally destroyed by fire?

3. A house is insured in three companies. One fourth the insurance is carried by Company A, at 20¢ per \$100; five eighths is carried by Company B at $1\frac{1}{2}\%$; and the remainder is carried by Company C at $1\frac{1}{4}\%$. What is the total cost of insurance if the insurable value of the house is \$10,000?

4. A building is insured for 1 yr. at 45¢ per \$100, insurable value \$15000. How much of the premium must be returned by the company if the insurance is canceled at the expiration of four months (1) by the company? (2) by the insured? (The short rate for 4 mo. is 70% of the annual premium.)

5. A house $50' \times 40' \times 40'$, costing 10ϕ per cubic foot, with an addition $14' \times 16' \times 10'$, costing 5ϕ per cubic foot, was insured for 1 yr. at 50ϕ per \$100. At the end of 60 da., the policy was canceled by the owner. What amount was returned to the insured? (The short rate for 60 da. is 30 % of the annual premium.)

6. A house valued at \$10,000 is insured in one company for \$3000, in another for \$2000. A loss of \$8000 is sustained. Under a coinsurance clause, how much loss must be borne by the owner?

7. A company carries insurance on its stock of merchandise, valued at \$25,000, to the amount of \$12,500 under a policy containing the coinsurance clause. In the event of a \$4850 loss, what will the company receive from the insurance company?

LIFE INSURANCE

311. In **life insurance**, the contingency insured against is the death of the insured. The policy may, however, agree to pay a fixed sum at the expiration of some specified time.

On the death of the insured, the amount of the policy is paid to some one named in the policy, called the **beneficiary**.

312. Life insurance policies may be classified as whole-life, term, and endowment.

313. In **whole life policies** the face of the policy is payable at the death of the insured.

314. In **term policies** the sum insured is payable only if death occurs within a stated period; after this period the policy lapses.

315. An **endowment policy** provides for the payment of a stated sum to the insured at the expiration of a stated period, if he is then living; or to his beneficiary in case of death before that time.

316. Rates for \$1000 insurance.

AGE	WHOLE-LIFE			ENDOWMENT		
	Ordinary Life	10-payment Life	20-payment Life	10 Years	15 Years	20 Years
21	\$ 19.16	47.26	29.22	103.01	65.34	47.73
22	19.56	47.98	29.66	103.10	65.94	47.83
23	19.98	48.72	30.13	103.19	66.04	47.94
24	20.42	49.48	30.64	103.29	66.15	48.06
25	20.88	50.27	31.14	103.39	66.26	48.18
26	21.36	51.09	31.67	103.49	66.38	48.31
27	21.87	51.95	32.24	103.60	66.50	48.45
28	22.41	52.83	32.81	103.73	66.63	48.61
29	22.98	53.72	33.39	103.86	66.78	48.78
30	23.58	54.66	34.01	103.99	66.94	48.96
31	24.21	55.62	34.65	104.13	67.11	49.16
32	24.88	56.63	35.32	104.29	67.29	49.37
33	25.59	57.66	36.01	104.46	67.48	49.60
34	26.34	58.73	36.73	104.64	67.69	49.85
35	27.13	59.85	37.49	104.84	67.92	50.12
36	27.99	61.00	38.28	105.05	68.17	50.42
37	28.85	62.19	39.11	105.28	68.44	50.75
38	29.79	63.43	39.98	105.53	68.73	51.11
39	30.78	64.71	40.87	105.80	69.05	51.51
40	31.83	66.03	41.83	106.09	69.41	51.95

The above is taken from the rate book of one of the large companies.

It is advisable to secure sample policies for examination of cash values, extended insurance, etc.

EXERCISES

1. Find the annual cost of an ordinary whole-life policy for \$3000 at the age of 25 yr.

According to the table the rate per \$1000 at age 25 is \$20.88. On \$3000 the premium (cost) is $3 \times \$20.88$, or \$62.64.

2. What is the annual cost of a \$10,000 policy on the life of a man 30 yr. of age on the ordinary whole-life plan?

3. A man, age 33, insures for \$5000 on the 15-year endowment plan. At the expiration of the period, how much more than the face of the policy will he have paid the company in premiums?

4. How much would the payments in the preceding problem amount to if the policy had been on the 20-year endowment plan?

5. A man 30 yr. of age took two policies; one for \$1000 on the 20-payment life plan, the other for \$1000 on the 20-year endowment plan. At the expiration of the 20 yr. the value of the 20-payment life policy was \$555; the value of the endowment policy was \$1000. As an investment, which was the better policy and how much? (Make no allowance for interest.)

6. A merchant 34 yr. old took out \$10,000 of ordinary life insurance as a means of protection to his creditors. What premium does he pay?

7. A 15-yr. endowment policy for \$2000, written on the life of a man 37 yr. old, is deposited as collateral for a loan of \$400 at 7%. What is the cost of premium and interest annually?

8. A man 29 yr. old pays a premium of \$207.72 on a 10-yr. endowment policy, and \$133.56 on a 20-payment life policy. How much insurance does he carry?

STOCKS AND BONDS

STOCKS

317. **Stock companies** are associations of persons authorized by law to transact business as one individual. They are called incorporated companies, or corporations.

318. **Stocks** represent the capital, property, etc., which a company invests in a business under the law of the state.

319. A **share** represents a certain equal part of the capital stock. Shares are of any value assigned to them, as \$25, \$50, \$100.

320. A **stock certificate** is a printed statement, signed by the officers of an incorporated company, stating the number of shares owned, the par value of each, etc., to which the holder is entitled.

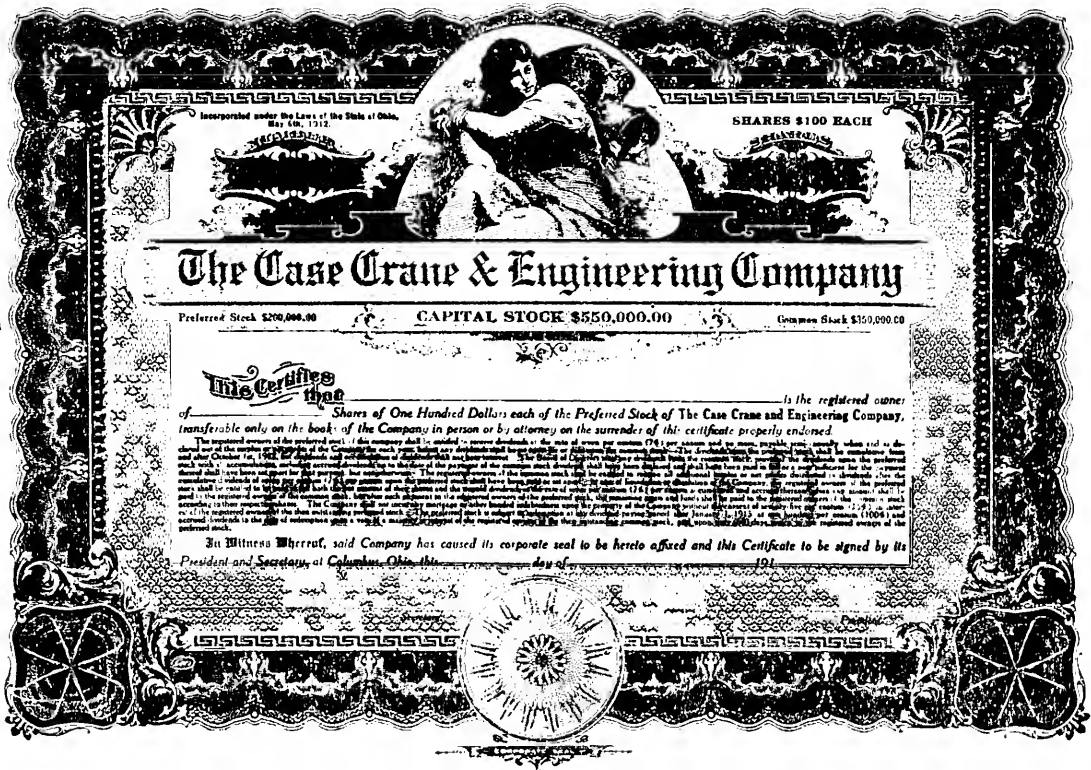
321. The **par value** is the value assigned to the stock by the corporation.

322. The **market value** is the value for which stock can be sold. It depends upon the dividends paid, security of the business, business conditions, etc. The market value is the quoted value, and may be *par*, *above par*, or *below par*. Thus, par being \$100, a stock quoted at 123 is above par; at 76 it is below par.

323. A **dividend** is a certain amount of profit, based upon the par value of the stock, divided among stockholders.

324. An **assessment** is a sum levied upon stockholders to make up losses, to make improvements, etc.

325. Preferred stock is stock to which some preference has been given, over other stock issued by a corporation. It



A CERTIFICATE OF STOCK

receives the first profits distributed, sometimes at a guaranteed rate per cent.

326. Common stock is the ordinary stock of a corporation, and carries with it no guarantee of dividends.

NOTE. Some corporations issue only one kind of stock, common stock; other corporations issue two or more kinds, as, preferred stock and common stock, or first preferred stock, second preferred stock, and common stock.

327. A stockbroker is a person who makes a business of buying and selling stocks for others; he usually charges, as **commission**, or **brokerage**, $\frac{1}{8}$ of 1% of the par value of the stock on each transaction. In some small exchanges the charge is $\frac{1}{4}$ of 1%. $\frac{1}{8}$ % will be taken as the charge in all problems in this book unless otherwise stated.

EXERCISES

1. A manufacturing company capitalized at \$12000 declares a dividend of $16\frac{2}{3}\%$. Find the dividends of a stockholder who bought 20 shares of stock at the par value of \$100.

$$16\frac{2}{3}\% = \frac{1}{6}.$$

$$20 \times \$100 = \$2000, \text{ stockholder's investment.}$$

$$\frac{1}{6} \text{ of } \$2000 = \$333\frac{1}{3}.$$

2. An investment of \$112,420 was made in Northern Pacific at $127\frac{5}{8}$. The stock pays regular dividends of 7%. Find the dividends.

$$127\frac{5}{8} + \frac{1}{8} \text{ (brokerage)} = 127\frac{3}{4}, \text{ cost per share.}$$

$$112,420 \div 127\frac{3}{4} = 880 \text{ shares.}$$

$$\$100 \times 880 \times .07 = \$6160, \text{ dividends.}$$

(Dividends are always figured on the par value.)

3. A man invests in Amalgamated Copper at 83, including brokerage. If this stock pays 5%, what per cent of income will he get from an investment of \$1992?

$$1992 \div 83 = 24 \text{ shares.}$$

$$\$100 \times .05 = \$5, \text{ dividend per share.}$$

$$\$5 \times 24 = \$120, \text{ income.}$$

$$120 \div 1992 = 6\frac{2}{83}\%.$$

Find the investment in each of the following, par value being \$100 and brokerage being $\frac{1}{8}$ of 1% on the par value.

	QUANTITY	STOCK	PRICE	COST		COM.	TOTAL COST	
4.	40	Ches. & Ohio	$69\frac{1}{4}$	\$2770		\$5	\$2775	00
5.	60	Mo. K. & T.	$27\frac{5}{8}$					
6.	120	Union Pac.	$169\frac{3}{8}$					
7.	74	Erie R. R.	$33\frac{3}{4}$					
8.	150	Gen. Elec.	160					
9.	24	Erie 2 pfd.	42					
10.	200	U. S. Steel	70					
11.	31	Nat'l Bis.	130					
12.	8	Amal. Copper	$78\frac{1}{2}$					
13.	65	Am. C. & F.	54					

14. A stock of par value of \$25 pays a yearly dividend of \$4.28 per share. What is the rate of income from this stock?

15. What can I afford to pay for a stock paying annual dividends of \$7.35 per share in order to net $5\frac{1}{2}\%$ on the investment? (No brokerage.)

Fill in the necessary amounts in the following:

	QUANTITY	STOCK	PRICE	COST		COM.	TOTAL COST	
16.	50	Amer. Can	$41\frac{1}{2}$	—		—	—	
17.	—	Diamond Match	107	\$5350		—	—	
18.	—	People's Gas	$117\frac{1}{2}$	—		\$2.50	—	
19.	100	Kan. C. & S.	—	—		—	\$2637	50
20.	—	Ill. Cent.	$128\frac{5}{8}$	1543	50	—	—	
21.	—	N. & W.	—	3227		3.50	—	
22.	35	C. M. & St. P.	—	—		—	3731	88
23.	—	West. Union	—	—		12.50	6137	50
24.	—	Quaker Oats	375	—		—	45015	
25.	—	D. L. & W.	—	6932		—	6934	

26. Find the income on an investment of \$4340 in Baltimore & Ohio R. R. at $108\frac{3}{8}$, brokerage $\frac{1}{8}$, if the stock pays 5% dividends.

27. If the money in problem 3 was borrowed through a bank at 5%, what was the income?

28. A man buys 50 shares of Union Pacific common at $172\frac{1}{8}$, and sells them at $175\frac{3}{8}$. If the broker charges $\frac{1}{8}\%$ on all transactions through his office, what was the man's gain?

29. If P., C., C., & St. L. pays 7% dividends, what investment, through a broker, at $109\frac{7}{8}$, brokerage $\frac{1}{8}\%$, will pay an annual income of \$2450?

30. A broker owns 300 shares of National Biscuit Co. stock which pays 8% annual dividends. He sells at $137\frac{1}{2}$

and invests the proceeds in U. S. Steel pfd., paying $6\frac{1}{2}\%$, at $111\frac{3}{4}$. Is his income increased or diminished and how much? (No brokerage.)

31. Find the cost, including brokerage, of 450 shares of stock paying 5% dividends on a par value of \$100 per share, if the income from the investment is 4%.

32. An investor can buy Pullman Co. stock paying 12%, at $160\frac{1}{4}$, or Pittsburgh Coal pfd. paying 8%, at $90\frac{1}{2}$. He chooses 500 shares of the former stock. Is his income more or less, and how much, than it would be on 500 shares of Pittsburgh Coal, brokerage $\frac{1}{8}\%$ on each transaction?

BONDS

328. A **bond** is an obligation to pay a certain sum of money at a stated time, with a fixed rate of interest payable at regular intervals. Interest payments are usually made semiannually, as Jan. 1 and July 1 or June 1 and Dec. 1.

Bonds are named for the government, corporation, city, etc., issuing them; for the time they are run, as 10-yr., 20-yr., etc.; for the security they offer, as first-mortgage, extension, refunding, etc.

Spanish War 3's were bonds issued by the government during the Spanish War, bearing 3% interest. Other titles, indicating the kind of bond, are: Panama Canal 2's,

Cleveland City 4's,
Denver Gas 5's,
Valley Traction
5's, etc.

329. A **coupon bond** is a bond having certificates or coupons attached,



COUPON FROM A COUPON BOND

United States of America.

The Ohio Gravel and Sand Company

Five Hundred Dollar Gold Bonds

For value received, The Ohio Gravel and Sand Company, a corporation duly organized under the laws of the State of Ohio, promises to pay to the bearer, or in case this bond shall be registered as hereinafter provided, then to the registered owner hereof, on or before the 1st day of December, 1917, Five Hundred (\$500.00) Dollars in Gold coin of the United States, of the present standard of weight and fineness, with interest thereon in like gold coin, from the first day of December, 1912, at the rate of seven (7) per centum per annum, payable semi-annually, on the 1st day of June and December, of each year, to the bearer of the annexed coupons.

The principal and interest of this bond are payable at the office of The Citizens Trust and Savings Bank, Columbus, Ohio, upon the surrender of this bond and the annexed coupons as they severally mature. This bond is one of a series of fifty (50) bonds, all of like tenor, date and amount, numbered consecutively from one (1) to fifty (50), both inclusive, all of which, with interest coupons attached, without preference or priority among them, and without distinction between principal and interest, are secured by a certain first mortgage of even date herewith, duly authorized, executed and delivered by The Ohio Gravel and Sand Company to Lowry F. Sater, as Trustee, conveying to said Lowry F. Sater, certain lands and properties therein referred to and described, in trust, to secure the payment of all said bonds and interest aforesaid.

Said bonds are entitled to the benefit of a sinking fund as provided in said mortgage deed, and subject to redemption either from the sinking fund or otherwise, in their serial numbers, commencing with the lowest number outstanding, at any interest bearing period, on or before the first day of December, 1917.

All redemptions prior to December 1, 1917, at the rates and premiums aforesaid, shall be optional with said promisor and shall be made only upon its written direction given and delivered to the trustee at least two (2) months before the date of the redemption next following such notice, and thereafter, the sinking fund shall be applied by the trustee to the redemption of bonds at their face value and accrued interest without such written direction.

If any default shall be made in any of the payments of interest on this bond, or in the payments into the sinking fund as provided in said mortgage, or in the payment of any taxes or assessments lawfully made or levied upon the whole or any part of the property made security hereof, and if any such default shall continue for a period of six (6) months after written notice thereof to said trustee, or by said trustee, or by the holder of this bond, or of any of its interest coupons then due and unpaid, then (and in any such event, at the option of the holder hereof, this bond and accrued interest shall thereupon become due, and payable.

This bond may be registered in the owner's name on the books of said The Ohio Gravel and Sand Company, at the Hartman Building, in the City of Columbus, Ohio, such registry to be noted on the bond by the company's registration agent, after which no transfer will be valid unless made on the books of said company by the registered owner or his attorney, and similarly noted on this bond, but the same may be discharged from registry by being duly transferred to bearer, after which it shall be transferable by delivery, and it shall continue subject to successive registrations and transfers to bearer as aforesaid, at the option of each holder. Such registry of this bond shall not restrain or affect the negotiability of the attached interest coupons by delivery merely, but unredeemed coupons may be surrendered at any time, and the installments of interest made payable thereafter only to the registered owner of this bond, such surrender to be noted on the bond by the company's registration agent. This bond shall be considered issued only when the trustee's certificate hereon is duly executed by Lowry F. Sater, as Trustee.

In Witness Whereof, the said The Ohio Gravel and Sand Company has caused these presents to be signed by its President and its corporate seal to be annexed, attested by its Secretary, and the attached coupons to be authenticated by the fac simile of the signature of its Secretary engraved thereon, this 1st day of December, 1912.

THE OHIO GRAVEL & SAND COMPANY.

Attest:

By

President.

Secretary.

A BOND

(197)

showing the amount of interest, when due, and where payable. There is one coupon for each interest payment, to be detached and negotiated like any other commercial paper.

330. A **registered bond** is a bond payable to the owner whose name is registered in the books of the issuing company. A registered bond can be transferred only by assignment.

331. The value of a bond depends on the security back of it, on the rate of interest it pays, and on the time it has to run. Values are affected by current money rates just as any other values are affected by them.

20 YEARS—Interest Payable Semi-annually.							
Net per An-num.	BONDS BEARING INTEREST AT THE RATE OF						
	7%	6%	5%	4½%	4%	3½%	3%
4	141.03	127.36	113.68	106.84	100.00	93.16	86.32
4.10	139.32	125.76	112.20	105.42	98.64	91.86	85.09
4½	138.90	125.37	111.84	105.07	98.31	91.54	84.78
4.20	137.63	124.19	110.75	104.03	97.31	90.59	83.87
4¼	136.80	123.42	110.04	103.35	96.65	89.96	83.27
4.30	135.98	122.65	109.33	102.66	96.00	89.34	82.68
4¾	134.75	121.51	108.27	101.65	95.04	88.42	81.80
4.40	134.35	121.14	107.93	101.32	94.72	88.11	81.51
4½	132.74	119.65	106.55	100.00	93.45	86.90	80.35
4.60	131.16	118.18	105.19	98.70	92.21	85.72	79.22
4¾	130.77	117.82	104.86	98.38	91.90	85.42	78.94
4.70	129.61	116.74	103.86	97.43	90.99	84.55	78.11
4¾	128.84	116.02	103.20	96.80	90.39	83.98	77.57
4.80	128.08	115.32	102.55	96.17	89.79	83.40	77.02
4¾	126.95	114.27	101.59	95.24	88.90	82.56	76.22
4.90	126.58	113.92	101.27	94.94	88.61	82.28	75.95
5	125.10	112.55	100.00	93.72	87.45	81.17	74.90
5.10	123.65	111.20	98.76	92.53	86.31	80.09	73.86
5½	123.29	110.87	98.45	92.24	86.03	79.82	73.61
5.20	122.22	109.87	97.53	91.36	85.19	79.02	72.85
5¼	121.51	109.22	96.93	90.78	84.64	78.49	72.34
5.30	120.81	108.57	96.33	90.21	84.09	77.97	71.85
5¾	119.77	107.60	95.44	89.38	83.27	77.19	71.11
5.40	119.42	107.28	95.14	89.07	83.01	76.94	70.87
5½	118.06	106.02	93.98	87.96	81.94	75.92	69.90
5¾	116.38	104.47	92.55	86.59	80.64	74.68	68.72
5¾	114.74	102.95	91.15	85.28	79.38	73.46	67.57
5¾	113.13	101.46	89.78	83.95	78.11	72.27	66.43
6	111.56	100.00	88.44	82.66	76.89	71.11	65.33
6¼	108.50	97.17	85.84	80.18	74.51	68.85	63.19
6½	105.55	94.45	83.34	77.79	72.24	66.69	61.14
6¾	102.72	91.83	80.95	75.50	70.06	64.62	59.17
7	100.00	89.32	78.64	73.31	67.97	62.63	57.29

The usual par value, called *denomination*, of a bond is \$1000. No fractional part of a bond can be purchased.

The following table, called a **bond table**, is used in determining the value of a bond, which has 20 yr. to run.

The left-hand column gives the rate of income; the top row gives the rate paid on the bond; the figures in the body of the table show what can be paid for any given bond to net a certain per cent on the investment. Thus, a 5% bond, if bought for \$113.68, will net 4%; if bought at \$110.04, will net 4½%, etc.

Likewise, a 3% bond, bought at \$84.78 will net 4½%.

Also, a man wishing to make an investment on a 5½% basis might buy a 4½% bond at \$87.96.

EXERCISES

Allow for no brokerage in the first seven problems.

1. Using the table, find the price at which 20-yr. bonds can be bought to yield as follows:

5% bond to yield $4\frac{1}{2}\%$.

6% bond to yield 5.4%.

$4\frac{1}{2}\%$ bond to yield 5.3%.

2. Find the rate of income on 20-yr. bonds bought as follows:

6% bonds bought at 114.27.

$3\frac{1}{2}\%$ bonds bought at par.

5% bonds bought at 80.95.

3. A man invested in first-mortgage industrial bonds bearing 5% at a price which yielded him 4.80%. What was the quoted price?

(In this problem, and in those to follow, consider all bonds as maturing in 20 yr.; interest coupons as due Jan. 1 and July 1.)

4. An issue of \$15000 of 5% bonds is sold for \$14,449.50. What rate do they pay the purchaser?

5. On June 12, I purchased five \$1000 $3\frac{1}{2}\%$ bonds quoted at 82 and interest. How much do I pay for them?

SUGGESTION. Compute the accrued interest from the time of the last payment, and add to the quoted price.

6. If I buy a \$1000 7% bond at 102.8 + \$27.50 accrued interest, what is the rate of income on my investment?

7. A municipal loan of \$41,000,000 3% bonds was oversubscribed 70 times, that is, the subscription amounted to \$2,870,000,000. If the bonds were allotted pro rata according to subscriptions, what amount was awarded to the man who subscribed for \$130,000 worth?

8. A man having \$7000 to invest wishes to secure an income of $4\frac{5}{8}\%$ on a $4\frac{1}{2}\%$ bond. How many \$1000 bonds can he buy, and how much will he have left of his \$7000?

NOTE. The broker's charge for the purchase or sale of a bond is the same as for stocks; viz., $\frac{1}{8}$ of 1%, or \$1.25 for each \$1000 bond.

Fill in the blank spaces in the following. Notice that under "quantity" the par value of the purchase is given instead of the number of bonds.

	QUANTITY	BOND	PRICE	COST		COM.	TOTAL COST
9.	\$6000	C. & O. $4\frac{1}{2}$'s	99 $\frac{1}{2}$	—		—	—
10.	9000	Cin. Sewer 4's	—	\$9225		—	—
11.	—	U. S. Reg. 2's 1930	—	—		\$15	\$12195
12.	5000	Scioto Val. 5's	—	4900		—	—
13.	—	U. S. Mex. 4's	—	8987	50	—	9000

14. A man deposits \$25000 with his broker for the purchase of Northern Pacific 4's at $98\frac{1}{4}$, brokerage $\frac{1}{8}\%$. How many bonds will be purchased for him, and what cash balance will he have left with the broker?

15. Cole & Co., brokers, sent this statement to R. T. Allen:

"We have this day bought for your account and risk as per instructions and in accordance with the rules and customs of the New York Stock Exchange, through Wilson & Co.:

QUANTITY	STOCK OR BOND	PRICE	COST	COM.	TAX	TOTAL COST
\$4000	A. E. & C. 5's	101 $\frac{3}{4}$	—	—		—
\$5500	Am. G. & E. 5's	103	—	—		— "

What is the amount of R. T. Allen's investment? What commissions do Cole & Co. get? What is the rate of income on the entire investment for the first year?

NOTE. The tax, called a transfer tax, is a charge, paid by the seller, on all stock sales. This charge is 2¢ per share of \$100 par value; the tax on shares of \$25 par value would be 50¢ per 100 shares.

EXCHANGE

332. **Exchange** is the term applied to the transfer of money from one place to another otherwise than by actually sending the money itself.

333. Exchange is **domestic**, if between two places in one country ; **foreign**, if between two places in different countries.

334. There are three important factors in exchange :

1. The amount to be transferred.
2. The charge for such transfer.
3. The value of ready money in one place as compared with that in another.

335. It is not surprising that money may have different values in different places. This value depends on the rate at which it can be borrowed from the banks. If the banks in one community have plenty of money on hand, and there is no great demand for it, money will be “cheap” ; *i.e.*, the rate of interest on loans will be low. But if there is no great surplus of cash on hand, and much money is needed for carrying on business, money will be “dear” or “tight” ; *i.e.*, the rate will be higher.

336. The value of money in one place as compared with its value in another is called the **rate of exchange** between these two places.

337. Rates of exchange fluctuate, being governed by trade, or business conditions. Published rates of exchange usually include the cost of the transfer.

338. The principal money centers in the United States are New York, Chicago, and San Francisco, and rates of exchange are usually quoted as "New York exchange," etc. Foreign exchange would be quoted as "London exchange," "Paris exchange," etc.

DOMESTIC EXCHANGE

339. Money may be transferred by :

(a) **Postal money order.** The government, through its post offices, accepts money and issues an order therefor, payable at any office the purchaser may desire. Postal money orders are issued for any amount up to \$100.

The cost of transfer by postal money order, called a "fee," is determined by the following table :

TABLE

For orders for sums not exceeding \$2.50	3 cents.
If over \$2.50 and not exceeding \$5.00	5 cents.
If over \$5.00 and not exceeding \$10.00	8 cents.
If over \$10.00 and not exceeding \$20.00	10 cents.
If over \$20.00 and not exceeding \$30.00	12 cents.
If over \$30.00 and not exceeding \$40.00	15 cents.
If over \$40.00 and not exceeding \$50.00	18 cents.
If over \$50.00 and not exceeding \$60.00	20 cents.
If over \$60.00 and not exceeding \$75.00	25 cents.
If over \$75.00 and not exceeding \$100.00	30 cents.

(b) **Express money order.** This is similar to a postal order except that it is issued by an express company instead of by the government.

The fee for an express money order is the same as for a postal money order.

(c) **Telegraphic money order.** Telegraph companies will accept money at any office and pay it out at any other office after an exchange of telegraphic messages.

Besides the charge for a fifteen-word message, the cost is: 25¢ for \$25 or less; 35¢ for over \$25 and not over \$50; 60¢ for over \$50 and not over \$75; 85¢ for over \$75 and not over \$100. After the first \$100, up to and including \$3000, add 25¢ for each \$100 or part thereof.

(d) Bank check. (See § 253.)

In local transactions involving bank checks there is no charge for collecting.

Collection is usually charged on bank checks sent to any place outside the city in which they are issued or outside the banking zone where they are received. The charges vary according to the amount of the check, the bank's arrangement with the customer, the distance between the two towns, etc.

(e) A certified check is a check across the face of which some officer of the bank, usually the cashier or paying teller, has stamped and signed a statement certifying that the check is good.

ARTHUR THOMPSON AND CO. LIMITED Birmingham, Ala.	Birmingham, Ala. <u>Mar 2</u> 191 <u>3</u> No. <u>328</u>	
	AMERICAN TRUST & SAVINGS BANK <small>61-12</small> <small>OF BIRMINGHAM, ALA.</small>	
	Pay to the order of	<u>Walter A. Bierg</u>
	<u>\$467⁵⁰</u>	
	<u>Four hundred sixty seven ⁵⁰/₁₀₀</u> Dollars	
<u>Arthur Thompson & Co. Ltd</u>		

A CERTIFIED CHECK

(f) Certificates of deposit (see § 252) are issued without charge, but are subject to the usual rates of exchange if cashed at a point other than that at which they are issued.

(g) A bank draft is a check drawn by one bank on another. It is the most common form of exchange because it is safe.

Travelers' checks are issued at a cost of $\frac{1}{2}\%$ of the amount with a minimum charge of 50¢.

(i) A **commercial draft** is a form of commercial paper used commonly in collecting accounts. It is an order, signed by the creditor or drawer, directing the debtor, or drawee, to pay a stipulated amount to a designated third party, usually a bank.

For example: A and B live in different cities, and A owes B \$100. B may "draw" on A for the amount, using a commercial draft for the purpose. B deposits this draft with his bank, which forwards it to a bank in the town where A does business. This bank presents the draft to A, and if it is honored (paid) by A, deducts its charge for collecting and remits the proceeds to B's bank. B's bank thereupon pays, or credits, the amount to B less its charge, if any, and the transaction is closed.

NO PROTEST	No. _____	The Scioto Sign Company	
	Kenton, Ohio, <i>Apr 11, 1913</i>		
	At Sight pay to the Order of <i>The Old National Bank</i>		
	<i>Five hundred twenty six</i> $\frac{8}{100}$ <i>00</i> — Dollars, \$ <i>526</i> $\frac{80}{100}$		
	<small>Value received, and charge to the account of</small>		
	To <i>Allen and Hughes</i> <i>Little Rock, Ark.</i>	The Scioto Sign Company By <i>P. L. Duffey</i>	

A COMMERCIAL DRAFT

The charge for collecting commercial drafts is not fixed by any rule, but is slightly larger than that on other kinds of exchange owing to the trouble of presenting and collecting.

Commercial drafts usually read "with exchange" or "without exchange." "With exchange" means that the collection charge is to be added to the face of the draft

when presented, and paid by the person drawn on; "without exchange" demands payment of only the face of the draft, and the charge is deducted after collection and the proceeds paid or credited to the maker of the draft.

340. When a draft is presented for payment, the person on whom it is drawn may, if he wishes, "accept" it, that is, write across the face of the draft the word "accepted," with the date, and sign his name. This **acceptance** acknowledges the indebtedness and the correctness of the amount, and virtually constitutes a promise to pay. The accepted draft is held until due, and then presented for payment.

341. A **protest** is a written or printed declaration by a notary public that a negotiable paper has been dishonored; *i.e.*, payment has been refused.

342. A **sight draft** is a commercial draft calling for payment "at sight" or at presentation.

343. A **time draft** is a commercial draft calling for payment at some future time, say in 3 da., 10 da., 30 da., etc.

344. The person drawn on may honor a commercial draft in two ways: by immediate payment, or by acceptance. (See § 340.)

345. Charges for exchange ought never to exceed the cost of shipping the actual money. In general they may be divided into two classes:

1. Those paid by the sender by the purchase of an order, payable at par at the point to which the money is sent. These include bank drafts, money orders, etc.
2. Those paid by the recipient of the money on the receipt of an order payable at the point from which the money is sent. These include bank checks, commercial drafts without exchange, etc.

346. New York, Chicago, and San Francisco being the principal money centers of the United States, most exchange is drawn against banks in those cities. Any large city, however, is a money center for the territory surrounding it, and banks in small towns issue drafts on banks in these various centers.

347. The rate of exchange (§ 336) is usually quoted in cents per \$1000 premium or discount. Thus:

At Detroit	New York exchange at par.
At Louisville	New York exchange at 20 ¢ premium.
At St. Paul	New York exchange at 45 ¢ premium.
At Denver	New York exchange at 80 ¢ premium.
At Chicago	New York exchange at 15 ¢ discount.

EXERCISES

1. What is the cost of a postal or express money order for \$1.12? for \$23.70? for \$75? for \$5.95? for \$17.25?

2. The fifteen-word telegraphic rate from Cincinnati to Dallas being 80 ¢, how much will it cost to send \$100 by telegraphic money order?

3. A, in Boston, sends to B, in Denver, his personal check for \$250. The charge for collection is $\frac{1}{10}\%$. How much does B receive on the check?

4. What is the cost of sending \$328.50 by express money order? \$125? \$157.50? \$418.33?

SUGGESTION. Distinguish between the cost of the order which includes the amount sent and the cost of sending.

5. A, in Cincinnati, wishes to send B, in Minneapolis, \$2456.87. If the collection charge is $\frac{1}{10}\%$, for how much must A draw his check to cover the amount plus the charge?

6. If New York exchange is quoted at 70 ¢ premium, how much will the following New York drafts cost: \$850, \$2378.60, \$1200, \$10000, \$375?

7. A bank charges 5 ¢ for each draft on Chicago of \$25 or less, 10 ¢ on \$25 to \$50, and 15 ¢ on \$50 to \$100. How much will each of the following drafts cost: \$7.55, \$23.61, \$87.19, \$66.90, \$40? What check will cover the cost of all the drafts?

8. An Indianapolis merchant owes a New York importer \$890.25. There is an agreement between them that payment is to be made in New York exchange, but that the importer is to pay the charge. If New York exchange is quoted at $\frac{1}{8}\%$ premium in Indianapolis, for what amount must the draft be made?

9. A certificate of deposit for \$500, issued by the City National Bank of Columbus, Ohio, Jan. 15, 1914, draws 3% interest. It is cashed in Milwaukee on Nov. 15, 1914, the collection charge being $\frac{1}{10}\%$. How much is paid for it by the Milwaukee bank?

10. How much will a person pay for 10 travelers' checks of \$20 each? for 10 of \$100 each?

11. If San Francisco exchange is quoted at $\frac{1}{20}\%$ discount, what will be the cost of drafts on a San Francisco bank for \$1800, \$5000, and \$1250?

12. On July 16, A drew on B for \$450 at 30 da. sight, without exchange, and placed the draft with his bank for collection. Allowing four days' time in the mails, on what date may A expect returns from the draft, and what will be the proceeds, if the collection charge is $\frac{1}{6}\%$?

13. A Cleveland manufacturer sells goods to a dealer in Decatur, Ill., on Nov. 1, amounting to \$2750.60, net 30 da. By agreement the manufacturer draws on the dealer on Dec. 1 at sight, with exchange. For what amount must the dealer draw his check to meet the draft, if the exchange (charge) is $\frac{1}{10}\%$.

14. Smith & Co., of St. Louis, sell Jones & Co., of Grand Rapids, Mich., goods amounting to \$1976.50, f. o. b. cars St. Louis. Smith & Co. draw on Jones & Co. at sight, less 2 % for cash. The bank's charge for handling the draft is $\frac{1}{8}$ %, to be paid by Jones & Co. How much will Jones & Co. have to pay in order to get the goods from the railroad company, the freight charges being \$72.41?

348. The **clearing house** is an organization of banks in a large city for the purpose of exchanging local checks.

1. Determine the balance for or against each bank in the following clearing house statement. Check your work.

BANK NO.	CR.		DR.		CR.		DR.	
1	\$ 123678	04	\$ 125919	21				
2	87664	13	82479	20				
3	108362	87	101475	78				
4	52117	69	63540	03				
5	72604	10	71012	61				

FOREIGN EXCHANGE

349. When money is sent from one country to another, the sending agent (post office, express company, or bank) converts the amount to be sent into terms of the money of the country to which it goes.

350. Money is thus transferred by: (a) postal money orders, (b) express money orders, (c) telegraphic or cable money orders, (d) travelers' checks, (e) letters of credit, (f) bills of exchange.

351. On money orders and travelers' checks, and on letters of credit, the **cost of exchange** is determined by pub-

lished rates. On bills of exchange it varies according to the money market, as determined by conditions of trade.

$\frac{1}{2}\%$ is usually charged for issuing travelers' checks.

The charge for cable transfer is the cable rate of exchange, plus $\frac{1}{4}\%$ commission, plus the telegraph and cable charges.

352. Letters of credit are issued by bankers. They call for the payment of a specified amount at any one of a number of banking institutions in various parts of the world.

The usual charge for issuing a letter of credit is 1%.

353. Bills of exchange include bank and commercial drafts, checks, etc., used in the transfer of funds from one country to another. They include :

(a) Those issued by banks. These are used in foreign exchange just as bank drafts are used in domestic exchange. They are sometimes issued in duplicate or triplicate, the various copies being sent by different mails to avoid mistakes and loss.

(b) Those issued by individuals. These include all forms of commercial drafts. They may be divided into two general classes : (1) *Documentary bills*, with shipping receipts, bills of lading, insurance policies, etc., attached as evidence that the drawer of the draft has performed his part of the transaction ; (2) *Clean bills*, having no documents attached.

354. As in domestic exchange the cost of foreign exchange should never exceed the cost of shipping the actual money. The banks, through which most of the exchanges are made either directly or indirectly, maintain the rates by actual shipments of money, usually gold.

355. The **par of exchange** between any two countries is based on the value of the pure metal (gold or silver) contained in the unit coin of any country, expressed in terms of the coin of another country. For example: The United

States dollar (as a unit of money) contains 23.22 grains of fine gold ; English pound sterling contains 113.0016 grains. The par of exchange between the United States and England is, therefore, $113.0016 \div 23.22$, or $\text{£ } 1 = \$4.8665$. For the values of foreign money see page 241. The most important exchange rates are quoted as follows :

England	£	= \$4.8665
France	\$1	= F. 5.18 $\frac{1}{2}$
Germany	M. 4	= \$.95
Holland	1 guilder	= \$.40

356. Indirect exchange (called arbitrage) is used when the quotations are such that it becomes cheaper to send money to London via Paris than to send it direct.

Suppose the quotations are as follows :

London exchange in New York	£ 1 = \$4.88 $\frac{1}{4}$
London exchange in Paris	£ 1 = F. 25.10
Paris exchange in New York	\$1 = F. 5.23

EXERCISES

1. I wish to send \$50 by money order to each of the following countries : Germany, England, France, and Holland. For how much will each order read, in terms of the money of the country to which it is to be sent ?

2. Exchange being at par, what will be the cost of a draft on London for £62 9s. 7d. ?

3. What will be the cost of a draft on Paris for F. 1000, exchange being quoted at $\frac{1}{20}\%$ premium ?

4. What will be the cost of a letter of credit for \$5000 ? Of travelers' checks for \$800 ?

5. What will it cost to cable \$400 to London, the cable charge being \$3 ? If London exchange is quoted at par, what amount in English money will be transferred ?

6. How large a draft can be bought on Denmark for \$2000? (See table in appendix.) On the Philippine Islands for \$125? On Sweden for \$285.67?

7. A firm owes £283 5s. 6d. in London, F.16,723 in Paris, and M.4933 in Germany. Quotations at the time of sending are as follows :

Sterling	\$4.8595
Francs	5.20
Marks94½

What will be the amount of the firm's check to purchase the exchange?

8. What will be the cost of a draft on the City of Mexico for 2550 pesos?

9. Will it be cheaper to send £1000 to London direct or to convert the money into Paris exchange and have it converted into pounds and sent to London? If so, how much cheaper? (Use quotations in § 356.)

10. What is the value of a bill of exchange on Paris for F. 7160, at 5.18½?

11. If exchange on Berlin is at par, what will be the cost of a bill of exchange for M. 3750?

12. How much English money should a traveler receive for a travelers' check for \$20? How much French money?

13. I hold a London acceptance for £112 8s. 10d., due in 45 da. If money is worth 4%, and the collecting charge is ¼%, how much can I get for the acceptance at my banker's?

14. How much German money would a traveler receive for a letter of credit for £200?

15. What will be the cost of a letter of credit for £100?

PARTNERSHIP

357. A **partnership** is an association of two or more persons for business purposes.

358. The **profits** or **losses** are shared according to agreement.

359. The **capital** of a firm is its total investment.

360. The **resources** of a firm are its total investment plus all debts or obligations owing to the firm.

361. The **liabilities** of a firm are the total debts or obligations to others.

362. The excess of the resources of a firm over its liabilities is its **present worth**; the excess of its liabilities over its resources is its **insolvency**.

363. The **net gain** is the excess of gains over losses.

364. The **net loss** is the excess of losses over gains.

365. To find what part of the profit each partner receives.

The capital employed in a business partnership is \$10,000, of which A furnishes \$6000, and B furnishes \$4000. What part of any profit should each receive?

A has invested $\frac{6}{10}$ ($\frac{6000}{10000}$) of the capital; hence he receives $\frac{6}{10}$, or $\frac{3}{5}$ of the profits.

B has invested $\frac{2}{5}$ of the capital; hence he receives $\frac{2}{5}$ of the profits.

If the profits should be \$2000, A would receive $\frac{3}{5}$ of \$2000, or \$1200; B would receive $\frac{2}{5}$ of \$2000, or \$800.

EXERCISES

1. A, B, and C form a partnership, agreeing to share gains and losses according to the amount of their investments. A invests \$4000, B \$5000, and C \$6000. Their gain the first year is \$1000. Find the present worth of each.

2. A and B have a joint capital of \$15000. At the end of 3 yr. their present worth is \$21000. What is each partner's gain if they share equally?

3. D invested \$4000 and E \$5000 in a business. At the end of one year their resources amount to \$18000, their liabilities to \$3000. Find each partner's present worth.

4. Three men invest \$3000, \$4000, and \$5000, respectively, in a business. They agree to share gains and losses according to investment. At the end of one year their present worth is \$8000. What was each man's loss?

5. Jones, Smith, and Doliver engage in business and agree to share gains and losses according to investment. Jones invests \$10,000, Smith \$12,000, and Doliver \$13,000. At the end of the first year their ledger accounts were as follows: Merchandise, gain \$10,000; patents, gain \$1000; expense, loss \$2500; real estate, gain \$1858.75; equipment, loss \$875; interest, loss \$200.50. What is each partner's present worth, and what per cent did he make on his investment?

6. On Jan. 1, 1911, A invests \$5000 in business. April 1, 1911, B puts \$4000 in the same business. Jan. 1, 1912, their ledger shows a gain of \$2000. How much does each partner gain, if they share according to the amount of the investment and length of time it was invested? [A invests \$5000 for 12 mo., which is the same as \$60,000 ($\5000×12) for 1 mo. B invests \$4000 for 9 mo., which is the same as \$36000 ($\4000×9) for 1 mo. Then A's share is $\frac{5}{8}$ of the profits.]

RAILROAD RATES

366. Railroad rates are of two kinds, class and commodity. **Class rates** are applied to all kinds of shipments, whether in carload lots or less, and the rates are fixed at a certain price per 100 lb. All articles of commerce are classified in each "freight territory" by a commission, which publishes the result of its work in the form of a freight classification. A joint rate commission, made up of representatives of all railroads in the territory, meets and fixes the rates to apply for each class, subject to the sanction of the Interstate Commerce Commission. These rates are published in what is known as a *freight tariff*.

367. **Commodity rates** are applied to certain kinds of heavy freight, usually in carload lots. Among the articles which get special commodity rates are : coal, coke, ice, rough stone, lumber, grain and grain products, iron ore, and articles of iron and steel manufacture. In the case of some of these the rate is based on 100 lb. as the unit, on others the rate is so much per barrel, while on most of them the ton is the basis for the rate. Among the commodities rated by the short ton (2000 lb.) are : rough stone, sand, ice, coal, coke, and paving brick. Iron ore and articles of iron and steel manufacture are rated by the long ton (2240 lb.).

368. Carload shipments must be loaded by the shipper and unloaded by the consignee, but the railroad company switches the car to whatever siding is most convenient. Less-than-carload shipments are handled by the employees of the railroads. Carload rates are always lower for the

same article than the less-than-carload rates, but are always based on a certain specified *minimum weight*; for instance, if a shipper has 25000 lb. of canned goods and the minimum weight is 30,000 lb., he will have to pay freight on 30,000 lb. in order to get the benefit of the carload rate. If he has over 30,000 lb., he pays freight on the actual weight.

EXERCISES

1. A shipment of sixth-class freight moved from Parkersburg, W. Va., to Syracuse, N. Y. The rate was $12\frac{1}{2}$ ¢ per 100 lb. and the consignment weighed 240,000 lb. What were the freight charges?

2. What is the fifth-class rate from Chicago to Richmond, Va., when a shipment weighing 4400 lb. costs \$118.80 for transportation?

3. The all-rail rate from Columbus, Ohio, to Providence, R. I., is $43\frac{1}{2}$ ¢ per 100 lb. for third-class; the rail-and-water rate between these points for the same class is 41 ¢ per 100 lb. How much will a man save by forwarding a third-class shipment weighing 12500 lb. by rail-and-water route?

SUGGESTION. Is it necessary to find full charges by separate routes?

4. In the official classification, automobiles by carloads are rated at 110 % of first-class rate, minimum weight 10,000 lb. The first-class rate between Vincennes, Ind., and Utica, N. Y., is 70 ¢ per 100 lb. What will it cost to forward a shipment of automobiles weighing 11,250 lb. between these two points?

5. A druggist in Bluefield, W. Va., received a consignment of toilet soap from Cincinnati, Ohio, rated at 35 ¢ per 100 lb. He notified the freight agent that there was an error in the freight rate, which should have been $27\frac{1}{2}$ ¢ per 100 lb. The agent refunded \$1.80. What did the soap weigh?

6. The rate on canned vegetables in less-than-carload lots, from Wheeling, W. Va., to Baltimore, Md., is 22¢ per 100 lb. The carload rate is fifth-class, or 15¢ per 100 lb., but the shipment is charged on a basis of 30,000 lb. minimum weight. A firm in Wheeling has a shipment of canned vegetables weighing 22,000 lb. Should it be billed as a carload lot, or less-than-carload shipment? How much cheaper is this way?

7. A shipment of first-class freight weighing 8000 lb. moved from Cleveland, Ohio, to Richmond, Va., via C. A. & C. and N. & W. Rys. The C. A. & C. received 20% of the rate and earned \$8.72 for hauling the shipment from Cleveland to Columbus, Ohio. What is the first-class rate from Cleveland, Ohio, to Richmond, Va.?

8. A consigned 40,000 lb. of baled hay to B at \$18 per ton, allowing B to deduct the amount of the freight charges from the invoice. A got \$330 from B. What was the freight rate?

9. The fourth-class rate from Columbus, Ohio, to New York is 27¢ per 100 lb., and the third-class rate is 38½¢ per 100 lb. By mistake the rate clerk applied the third-class rate to a shipment which should have been rated at fourth-class. The delivering road corrected the error and refunded the consignee \$16.10. How much did the shipment weigh?

10. The first four class rates from Buffalo, N. Y., to Seattle, Wash., are \$3.60, \$3.10, \$2.60, \$2.20 per 100 lb., respectively, governed by the western classification. This classification makes shoes first-class, leather in bags second-class, and last blocks fourth-class. What are the freight charges on the following shipment between these two points: 1 bx. shoes, 80 lb.; 1 bag leather, 120 lb.; 1 bbl. last blocks, 150 lb.?

11. The carload rate on flour from Minneapolis, Minn., to Tacoma, Wash., is 90 ¢ per 100 lb., minimum weight 56000 lb. The less-than-carload rate is \$1.83 per 100 lb. Washburn, Crosby & Co. have a consignment of 150 bbl. of Gold Medal flour to ship to Tacoma. Should they bill it as a carload or less-than-carload shipment? How much cheaper is this way?

12. A farmer in Cheyenne, Wyo., ordered a wagon, a hay rake, and a corn planter from a firm in Milwaukee, Wis. When these are taken apart and tied in bundles, the western classification makes wagons first-class, rakes second-class, and corn planters third-class. The wagon weighed 800 lb., the rake 350 lb., and the corn planter 500 lb. The rates for the first three classes, Milwaukee to Cheyenne, are \$3.05, \$2.60, \$2.20 per 100 lb. How much did he have to pay for his shipment?

13. The rate on flour carloads from Indianapolis, Ind., to Wilmington, N. C., is made $10\frac{1}{2}$ ¢ per 100 lb. to Kenova, W. Va., and 46 ¢ per bbl., Kenova to Wilmington. A carload of flour moved from Indianapolis to Wilmington. What were the transportation charges?

14. A farmer wants to move from Logansport, Ind., to Columbia, S. C., and has some live stock to ship with his household goods. The rate is made 35 ¢ per 100 lb. from Logansport, Ind., to Cincinnati, official classification, minimum weight 12000 lb.; and 53 ¢ per 100 lb. from Cincinnati to Columbia, S. C., southern classification, minimum weight 24000 lb. The shipment weighed 18000 lb. What are his freight charges for moving?

PARCEL POST

369. The Parcel Post Act of Congress, which went into effect on Jan. 1, 1913, provides for the transportation in the mails of merchandise at low rates.

The accompanying table gives the rates provided by law :

TABLE OF RATES

WEIGHT	FIRST ZONE		SECOND ZONE, 50 TO 150 MI.	THIRD ZONE, 150 TO 300 MI.	FOURTH ZONE, 300 TO 600 MI.	FIFTH ZONE, 600 TO 1000 MI.	SIXTH ZONE, 1000 TO 1400 MI.	SEVENTH ZONE, 1400 TO 1800 MI.	EIGHTH ZONE, ALL OVER 1800 MI.
	Local rate	Zone rate, 50 mi.							
1 lb.	\$ 0.05	\$ 0.05	\$ 0.05	\$ 0.06	\$ 0.07	\$ 0.08	\$ 0.09	\$ 0.11	\$ 0.12
2 lb.	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 lb.	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 lb.	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 lb.	.07	.09	.09	.14	.23	.32	.41	.51	.60
6 lb.	.08	.10	.10	.16	.27	.38	.49	.61	.72
7 lb.	.08	.11	.11	.18	.31	.44	.57	.71	.84
8 lb.	.09	.12	.12	.20	.35	.50	.65	.81	.96
9 lb.	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10 lb.	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11 lb.	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
20 lb.	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
50 lb.	.30	.54	.54						

The local rate is applicable to parcels intended for delivery at the office of mailing or on a rural route starting therefrom.

370. Under the parcel post act the country is divided into units. Each unit is a quarter of a quadrangle formed by meridians of longitude and parallels of latitude.

371. The average length of the quadrangles north and south is 69 mi., their average width east and west is 54 mi. The unit, therefore, is about $34\frac{1}{2}$ mi. long (north and south) and 27 mi. wide (east and west), with an area of a little over 930 sq. mi.

372. Each of the 6000 (approximately) units thus established is the center of a system of eight **zones**. The first zone, called the 50-mi. zone, consists of the unit itself and its eight contiguous units.

373. The limit of weight for packages for local delivery and for delivery at other post offices within the first and second zones is 50 lb. The limit of weight for delivery in other than the first and second zones is 20 lb.

EXERCISES

1. What will it cost to send a 7-lb. package to a point in the 300-mi. zone? One weighing 9 lb.?

2. How far (into what zone) can a package weighing 3 lb. be sent for 25 ¢? for 15 ¢? for 36 ¢?

3. A mail-order house sent out in one day 820 2-lb. parcels. 556 of these went into the 50-mi. zone, 234 into the 150-mi. zone, and 30 into the 300-mi. zone. What postage was paid on all of them?

4. How much will it cost to send a 20-lb. package 95 mi.?

5. Find the cost of sending 5 11-lb. packages into the fifth zone.

MISCELLANEOUS PROBLEMS

GROUP 1

Find the tax on the following :

1. \$8765 at .0091.
2. \$2300 at 4 mills on the dollar.
3. \$1400 at .0126.
4. \$6800 at \$1.75 per \$100.
5. \$3750 at 1.871 %.
6. The assessed valuation of a town is \$3,250,000, and a property owner pays \$44.85 on property assessed at \$3900. What is the total amount raised by taxation?
7. A's property, assessed at \$16000, is taxed 11 mills on the dollar. By paying his taxes promptly he secured a discount of 1 % from the amount. How much does he pay?
8. What is the duty on 2200 T. of hay at \$4 per ton? on 5000 lb. of writing paper at 3¢ per pound? on a yard of silk ribbon worth 40¢ at 50 %? on a pair of shoes costing \$3 at 15 %?
9. What is the duty in United States money on French goods valued at F. 11630 at 30 %?
10. What is the premium on a life insurance policy for \$2500, if the premium on \$10,000 is \$267.10?

GROUP 2

1. A building insured for \$11,000 is valued at \$16,500. Under the coinsurance clause, what will the insurance company pay in the event of a loss of \$1376.28?

2. A man has two houses worth, respectively, \$4500 and \$3200. Taxes are $1\frac{1}{2}\%$, insurance is \$15.50 on the first, and \$11.20 on the second. The first rents for \$40 per month; the second, for \$30. Which is the better investment?

3. An insurance company collects premiums amounting to \$14,692,368.32; it pays out losses amounting to \$5,342,200.20. What is the loss ratio (percentage of losses to premiums)?

4. The owner of a 1913 model 20-horse-power automobile costing \$2500 insures the machine (1) against injury to persons, at \$1.20 per horse power; (2) against damage by collision, at 2% of the value of the car; (3) against damage to property, at 30¢ per horse power; (4) against fire and theft, at $2\frac{1}{2}\%$ of the value of the car. What is the total amount paid by the owner for such protection?

5. A coal mining company whose yearly pay roll is \$30,000 is insured against claims for damage due to injury to its employees at \$1 per \$100 of the pay roll. What premium does the company pay?

6. What is the cost of 20 shares of stock at $55\frac{1}{2}$, brokerage $\frac{1}{4}\%$?

7. C buys 50 shares of stock at 106. In 18 mo. time he receives 4 dividends of 6% each. If money is worth 5%, how much does he gain over and above this rate?

8. A sent 70,000 lb. of wheat to be sold and the proceeds invested in Southern Pacific stock. The agent sold the wheat for 90¢ per bushel and charged a 2% commission. Southern Pacific cost 110, brokerage $\frac{1}{8}\%$. How many shares did A receive, and what surplus had he left over?

9. What is the rate of investment on Atlantic and Pacific Railroad 4's bought at 83, brokerage $\frac{1}{8}\%$?

10. I receive \$375 every 6 mo. as interest on Erie 5's. If I bought them at $98\frac{5}{8}$, plus $\frac{1}{8}\%$ brokerage, how much did I invest?

GROUP 3

1. The following is the form in which a broker keeps his records of a transaction made for a customer. In this account, the customer bought 100 shares of New York Central on Nov. 5, at 115. He deposited \$1000 on the transaction, the broker lending him at 6% the amount necessary to make the purchase. When the stock is sold, the customer is credited with the amount of the sale, interest is charged, and the balance is credited to the customer.

DR.				CR.			
Nov. 5	100 N. Y. Cent. 115	\$ 11500		Nov. 5	Cash	\$ 1000	
	Com. buying	12 50					
Nov. 20	Com. selling	12 50		Nov. 20	100 N. Y. Cent. 117	11700	
	Tax selling	2			Int. on \$1000 Nov.		
	Int. on \$11,512.50				5 to Nov. 20	2 50	
	Nov. 5 to Nov. 20	28 78					
	Balance	1146 72					
		\$ 12702 50				\$ 12702 50	

The profit in the transaction is the customer's balance (\$1146.72) minus his cash deposit (\$1000), or \$146.72.

NOTE. Interest is charged on the full amount of the purchase, and credited on cash deposit. For tax (\$2) see note, page 200.

Using the above as a model, make and fill in forms for the following transactions. Compute interest at 5%. Tax, 2¢ per share on sales.

DATE BOUGHT	STOCK	PRICE	AMOUNT	CASH DEP.	DATE SOLD	PRICE	AMOUNT
2. Aug. 3	20 P. Coal	23½		\$ 100	Oct. 18	25½	
3. May 10	100 U. S. Steel	75¼		800	Sept. 20	78	
4. Jan. 14	50 Gen. Elec.	180		1200	Feb. 19	192	
5. Mar. 2	75 Amer. Loco.	41½		375	Mar. 22	41½	
6. Dec. 10	200 So. Pac.	109½		5000	Dec. 20	111	

7. A bank bid \$831.58 above par for an entire issue of \$15000 5% school bonds. What return does the investment make the bank?

8. A corporation has outstanding \$6,000,000 of 6% preferred, and \$12,000,000 common stock. Its net earnings for the year are \$1,131,885. What per cent dividends will the common stock earn?

9. A man having \$3000 to invest buys 12 shares of stock at 400, borrowing enough additional on the stock as collateral to make the purchase. If he pays 5% on his loan, and the stock pays 25% dividends, what per cent is he making on his investment? (Par value, \$100.)

10. Which is the better investment, 100 shares of 6% preferred stock at $107\frac{3}{8}$, or 50 shares of 12% common at $214\frac{3}{4}$? (Brokerage, $\frac{1}{8}\%$.)

GROUP 4

1. A salesman sells in one year \$128,466.73 worth of goods on a 5% commission. His expenses for the year are \$3246.80, and he pays \$117.43 taxes and \$245.48 insurance. He buys 10 shares of bank stock direct from the owner at \$165 per share, and sets aside \$263.63 for a checking account at the bank. If he deposits the rest of his year's income in a bank at $3\frac{1}{2}\%$, what interest will it bring him annually?

2. I discount a 10-day acceptance for \$2500 at $5\frac{1}{2}\%$ and use the proceeds in buying a telegraph money order. If the 15-word rate is 80¢, what amount is available to the person receiving the order?

3. A draws on B at sight for \$162.57 with exchange. To meet the draft B borrows the money at his bank at 6% for 60 da. What is the amount of B's note when due? (Exchange, 20¢.)

4. A corporation issues \$75000 6% bonds, which it is forced to sell at 87. In paying the interest on its bonded

indebtedness what rate does it pay on the money actually received for the bonds?

5. A buys 100 shares of stock at $27\frac{1}{2}$ and sells them at 30. Allowing brokerage of $\frac{1}{4}\%$ on each transaction, how much does he gain?

6. The firm X, Y, Z makes a gross profit in one year of \$25,004. They charge off \$1484 to depreciation on machinery, \$675 to depreciation on trade mark, and pay \$6160 for general expenses and wages. If the three members of the firm are equally interested in the business, how much will each receive as his share of the net profits?

7. A borrows \$15000 on his note at 7%. To secure the note he takes out a life insurance policy for \$18000 at \$14.62 per thousand, annual premium. The \$15000 is invested in a third interest in a business which shows a profit at the end of the year of $33\frac{1}{3}\%$. What is the net rate of income on the investment?

8. Find the proceeds on the following sight drafts:

FACE OF DRAFT	COLLECTION AND EXCHANGE
\$28.63	10¢
162.45	$\frac{1}{4}\%$
4216.53	$\frac{1}{20}\%$
6.20	5¢
683.10	$\frac{1}{8}\%$
1370.64	$\frac{1}{10}\%$

9. Find the cost of a bill of exchange for :

£ 362 11s. 5d. if 1 £ = \$4.8725
F. 3624	if F. 5.16 $\frac{1}{4}$ = \$1
M. 11360	if M. 4 = \$.95 $\frac{1}{4}$

10. A bond issue of \$1,000,000 was oversubscribed $3\frac{1}{2}$ times; that is, the subscriptions amounted to \$3,500,000. If each subscriber receives bonds in proportion to his subscription, what did the subscriber receive who asked for \$14000 worth?

GROUP 5

1. How much will I receive for a 90-day acceptance for \$350, discounted at 6%, exchange $\frac{1}{4}\%$?
2. How much will it cost to telegraph \$125 from Cleveland, Ohio, to Nashville, Tenn., the telegraph rate being 60¢?
3. An importer buys £163 8s. 3d. worth of goods on which he pays a duty of 20%. For how much U. S. money must he sell to realize a profit of 20% of the cost?
4. A and B own a business, A holding a $\frac{3}{5}$ interest, and B a $\frac{2}{5}$ interest. If they sell a third interest for \$6200, what must A and B each receive in order that A, B, and C may thereafter be equally interested in the business?
5. A has \$5000 invested in a store, B has \$3000, and C has \$2000. They decide to add \$4000 to the capital employed. What part of this amount must each partner furnish in order that their relative interests may remain unchanged?
6. What is the value of a 90-day acceptance for £112 7s. 4d., London exchange being quoted at \$4.84 $\frac{1}{8}$, discounted at 6%?
7. An inventor holds 100 shares of 6% preferred stock, 200 shares of 5% common, and \$22000 6% bonds of the same company. What is his income?
8. A bank capitalized at \$300,000 has a surplus of \$317,578. What per cent dividends might be declared by appropriating 60% of the surplus to such a purpose?
9. A company purchased M. 18000 worth of German goods on which it paid \$450 duty and \$106.85 freight. The goods were sold for \$6320, less 2%. What was the profit?
10. A, holding B's note for \$270 with interest for 4 mo. at 7%, draws on B through the First National Bank of Burt, Iowa, at sight for the full amount. B's place of business is Denver, Col. Write the draft.

REVIEW PROBLEMS

ORAL EXERCISES

- | | | |
|--|---|--------------------------------|
| 1. 34×11 | 20. 325×925 | 39. $424 \div 25$ |
| 2. 45×200 | 21. 750×750 | 40. $7650 \div 50$ |
| 3. 93×97 | 22. 290×150 | 41. $60 \div \frac{4}{3}$ |
| 4. 472×300 | 23. 756×175 | 42. $725 \div 1.33\frac{1}{3}$ |
| 5. 67×11 | 24. 35×35 | 43. $125 \div \frac{3}{4}$ |
| 6. 4532×11 | 25. $.95 \times .65$ | 44. $452 \div .16\frac{2}{3}$ |
| 7. 6460×150 | 26. 8.5×750 | 45. $7840 \div 125$ |
| 8. 9540×125 | 27. $12\frac{1}{2} \times 1.25$ | 46. $475 \div \frac{5}{4}$ |
| 9. $6\frac{1}{2} \times 6\frac{1}{2}$ | 28. 75×25 | 47. $5896 \div .16\frac{2}{3}$ |
| 10. $5\frac{1}{2} \times 5\frac{1}{2}$ | 29. $10\frac{1}{4} \times 10\frac{1}{4}$ | 48. $.12\frac{1}{2} = ?$ |
| 11. 450×450 | 30. $6\frac{2}{3} \times 6\frac{2}{3}$ | 49. $.08\frac{1}{3} = ?$ |
| 12. 6.5×6.5 | 31. $5\frac{1}{4} \times 5\frac{1}{4}$ | 50. $.33\frac{1}{3} = ?$ |
| 13. $9\frac{1}{3} \times 9\frac{1}{3}$ | 32. $2\frac{3}{8} \times 2\frac{3}{8}$ | 51. $.87\frac{1}{2} = ?$ |
| 14. 425×625 | 33. 925×725 | 52. $.58\frac{1}{3} = ?$ |
| 15. 625×4.25 | 34. $8\frac{1}{3} \times 8.3\frac{1}{3}$ | 53. $1.33\frac{1}{3} = ?$ |
| 16. $8\frac{1}{3} \times 8\frac{1}{3}$ | 35. $\frac{1}{27\frac{1}{2}} \times 1375$ | 54. $.55\frac{5}{9} = ?$ |
| 17. $7\frac{3}{4} \times 7\frac{3}{4}$ | 36. $\frac{51}{63} \div 17$ | 55. $.56\frac{1}{4} = ?$ |
| 18. 425×8250 | 37. $5\frac{2}{3} \div 6$ | 56. $.06\frac{1}{4} = ?$ |
| 19. $9\frac{1}{4} \times 9\frac{1}{4}$ | 38. $\frac{2}{3}$ of $\frac{4}{7}$ | 57. $.62\frac{1}{2} = ?$ |

Find the following:

- | | | |
|-----------------|-------------------------------|------------------|
| 58. 10 % of 280 | 61. $16\frac{2}{3}$ % of 660 | 64. .2 % of 640 |
| 59. 20 % of 60 | 62. $108\frac{1}{3}$ % of 240 | 65. .3 % of 5 |
| 60. 48 % of 175 | 63. 92 % of 150 | 66. 150 % of 500 |

67. .1 % of .1 69. $.08\frac{1}{3}$ of 12 71. $83\frac{1}{3}$ % of $\frac{36}{7}$.
 68. .01 % of 740 70. $16\frac{2}{3}$ % of 45 72. $\frac{2}{3}$ % of 36 .
 73. $56\frac{1}{4}$ % of 48 74. 12 is 4 % of what number ?
 75. 24 is $41\frac{2}{3}$ % of what number ?
 76. .25 is 80 % of what number ?
 77. $.83\frac{1}{3}$ is 20 % of what number ?
 78. 4 is $\frac{5}{6}$ % of what number ?
 79. 35 is $6\frac{1}{4}$ % of what number ?
 80. $.62\frac{1}{2}$ is $83\frac{1}{3}$ % of what number ?
 81. Reduce $\frac{5}{8}$ to 24ths. 83. Reduce $\frac{64}{8}$ to lowest terms.
 82. Reduce $\frac{3}{7}$ to 21sts. 84. From $\frac{5}{6}$ take $\frac{2}{3}$.
 85. From $\frac{7}{4}$ take $\frac{3}{16}$.

Find the interest at 6 % :

	PRINCIPAL	TIME
86.	\$400	3 yr.
87.	250	120 da.
88.	325	80 da.
89.	720	1 mo. 10 da.
90.	600	40 da.

91. A man had \$2000 and spent 20 % of it. How much money had he left ?

92. Owning a farm of 420 A., I sold $16\frac{2}{3}$ % for \$4900. What price per acre did I receive ?

93. Owning 60 % of the stock of a manufacturing company, I sold 20 % of my share for \$2400. What was the value of the entire stock ? of my share ?

94. I sold a horse for \$200, which was at a loss of 20 % of the cost. Find the cost.

95. How many times can 36 be subtracted from 300 ?

96. A baseball team wins 14 games and loses 10. What is its per cent of games won ?

97. A catalogue lists an article at \$45, but allows 25 % and 20 % off. Find the net selling price.

98. Brown gained \$500 the first year and after adding his gain to his investment, lost $\frac{1}{4}$ of the amount the second year. He then had \$1800. How much had he at first?

99. A invests \$600 and B \$800. Their gain is \$300. What should each receive?

100. What will it cost to send a 7-pound parcel into the 150-mile zone by parcel post?

WRITTEN EXERCISES

1. Add and check :

2875648
1792559
1624076
9217845
7384957
6294738
7549629
6872597
827358
75646
198478
7354629
6473746
1897645
735490
6487
954398
2872594
7658
64352

2. Add and check :

9572674
8468739
7254927
6248725
9764523
78654
837642
9872937
1640735
8295439
764375
4874062
567432
28739
543721
17687
649586
7468929
8295437
7804

3. Add and subtract alternately :

8972549
2762597

2437538

7642973

297364

8549321

5432976

849278

4. Add :

687.25
— 384.20
72.85
16.92
140.62
45.07
83.98

5. Add by the balancing account method and check :

	BALANCE	CHECK	DEPOSITS	BALANCE
<i>A</i>	985.72	342.89	58.72	—
<i>B</i>	487.68	159.20	268.73	—
<i>C</i>	95.98	20.07	75.00	—
<i>D</i>	168.75	68.92	934.58	—
<i>E</i>	72.59	14.09	54.72	—
<i>F</i>	354.85	215.95	134.17	—
Total				

6. Find the balance :

Dr.	Cr.
\$2874.58	\$1975.62
1784.63	8573.74
8873.19	175.14
982.73	1354.05
<u>3214.95</u>	<u>938.15</u>

7. Multiply and check :

$$\begin{array}{r} 78549872 \\ \underline{927} \end{array}$$

8. Multiply :

$$\begin{array}{r} 147268 \\ \underline{9154} \end{array}$$

9. Find the G. C. D. of 24, 132, 144.

10. Find the L. C. M. of 81, 120, 117.

11. Add :

$$\begin{array}{r} 765.64\frac{1}{8} \\ 42\frac{1}{4} \\ 678.6\frac{1}{12} \\ 4572.68\frac{1}{3} \\ 62.14\frac{1}{6} \\ \underline{759\frac{3}{4}} \end{array}$$

12. Multiply $534\frac{3}{4}$ by $8.24\frac{1}{3}$.

13. Divide $782596\frac{15}{341}$ by $.16\frac{2}{3}$.

14. Multiply 18 bu. 3 pk. 2 qt. by 12.

15. Extend and foot the following :

NUMBER OF POUNDS	ARTICLE	PRICE	VALUE
84000	Hay	\$10 per ton	—
12578	Corn	80 ¢ per bushel	—
6250	Oats	40 ¢ per bushel	—
5348	Bran	\$18 per ton	—
28364	Chop	\$30 per ton	—
	Total . .		

16. The hypotenuse of a right triangle is 400 ft., the base 200 ft., what is its area?

17. Find the area of a circle whose diameter is 10 ft.

18. What will it cost to carpet a room $18' \times 16'$, if the carpet is $\frac{3}{4}$ of a yard wide, at \$1.50 per yard?

19. At \$5 per thousand what will it cost to shingle a flat roof $150' \times 60'$, allowing 850 shingles to a square?

20. How many cords of wood in a pile $20' \times 4' \times 8'$?

21. A commission merchant sold a carload of lumber, and sent the owner \$3564.25, after paying \$70 freight charges, and retaining $2\frac{1}{2}\%$ commission for selling. What was his commission?

22. A pay roll made from time slips and rates per hour as shown below will amount to how much?

	TIME	RATE	AMOUNT
<i>A</i>	24 hr.	.12 $\frac{1}{2}$ per hour	—
<i>B</i>	40 hr.	.14 per hour	—
<i>C</i>	48 hr.	.20 per hour	—
<i>D</i>	50 hr.	.17 $\frac{1}{2}$ per hour	—
<i>E</i>	40 hr.	.22 $\frac{1}{2}$ per hour	—
		Total . . .	—

23. What sum must a man invest in 5% bonds bought at 120 to give his son an annual income of \$1200?

24. A man having money to invest in bonds has a list from which he wishes to select the investment most profitable to him. The list includes :

1. 5 % bonds, market price \$110
2. 6 % bonds, market price 120
3. 4 % bonds, market price 100
4. $4\frac{1}{2}\%$ bonds, market price 60
5. 3 % bonds, market price 90

Which bonds should he purchase?

25. A commission merchant sold for his principal 400 bu. of potatoes at 90¢ per bushel, and after deducting \$3.80 for freight, and his commission of 3%, remitted a check for the proceeds. What was the amount of the check?

26. I sold two houses for \$8000 each. On one I gained 20 % of the cost, and on the other I lost 20 % of the cost. Did I gain or lose on the whole transaction, and how much ?

27. A real estate dealer sold a house for \$7000 ; after paying expenses of \$440, he finds that his gain is $16\frac{2}{3}$ % of the cost. How much did the house cost him ?

28. Which is better for the buyer, a discount series of 20, 20, and 10, or 25, 15, and 10, and how much ?

29. An article at \$4.50 was discounted 20 % and \$.80. What was the second rate of discount ?

30. How shall goods that cost \$45 be marked so that a discount of 10 % may be made from the marked price and then sell at a profit of 10 % of the cost ?

31. A lent B \$900 for 2 mo. At the same rate, what amount shall B lend A for 3 mo. in return for the accommodation ?

32. On April 3, a carriage manufacturer in Ohio sells a shipment to a Texas dealer, amounting to \$850. He attaches the bill of lading to a 60-day sight draft which he discounts at his bank at 6 %. Allowing 12 da. before the consignee can receive and inspect the shipment, and 3 da. for return to the Ohio bank after payment in Texas, what are the proceeds of the draft, collection charges being $\frac{1}{10}$ % ?

33. A shoe manufacturer discounts at his bank the following notes from out-of-town customers :

FACE	TIME	RATE %
\$ 123.45	90 da.	7 % from date
240.00	30 da.	$5\frac{1}{2}$ % from date
93.25	30 da.	6 % from date
107.70	60 da.	6 % from date

What are the proceeds of these notes, if the bank discounts them at 5 %? (Add 4 days' time in discounting to allow for time in transit.)

34. A farmer ships a car containing 38762 lb. of potatoes to a broker, who sells them for 82¢ per hundredweight. Freight charges are \$73.20, and the broker charges a commission of 2 %. What amount is returned to the farmer?

35. The *Detroit Times* shows the following circulation for one month:

MON.	TUES.	WED.	THU.	FRI.	SAT.	TOTAL
			36555	36302	36310	—
36258	36338	36303	36233	36500	36173	—
36147	36668	38710	40871	43005	43609	—
43458	43351	35835	35967	35881	35733	—
76940	36514	36359	36998	36198	36329	—

- Find the total circulation for each week.
- Find the total circulation for each day in the week.
- Find the total circulation for the month.
- What is the average circulation per day?
- What day is nearest the average?

36. Which of two salesmen, A and B, whose yearly sales are given below, returns more profit to his firm?

A sells \$225,000 worth, showing an average profit to the firm of $11\frac{1}{2}\%$ of his sales; cost of collections is $\frac{1}{4}\%$, selling expenses, \$6450.

B sells \$260,000 worth at an average profit of $10\frac{3}{4}\%$ of his sales; cost of collection, $\frac{5}{8}\%$, selling expenses, \$7200.

37. A company issues \$100,000 7 % preferred stock, and \$100,000 of common stock. It earns in one year \$28,762.20. What amount can be added to the surplus if it pays 11 % dividends on the common stock, and charges \$3333.33 to depreciation of plant and equipment?

38. If the advertising expense incident to the sale of an article is \$10,000, and this expense is divided among manufacturer, jobber, and retailer according to the profits of each, what part will be paid by each, if the profits are, respectively, 22 %, 13 %, and 15 % ?

39. A merchant buys \$15000 worth of goods at 2 %–10, net 60. How much will he save by borrowing the money for 30 da. at 6 % in order to discount the bill ?

40. If a retailer sells an article for \$12.50, showing 20 % profit on the selling price, at what can he afford to sell it if he is willing to take only 10 % profit on the cost ?

41. Find the difference between simple interest and compound interest on \$9375 for 10 yr. at 6 %.

42. An agent sold for his principal goods to the amount of \$10,000, charging 3 % commission. He invested the proceeds in wheat after deducting a commission of 2 % for buying. If he paid \$.90 per bushel for the wheat, how many bushels did he buy ?

43. The average circulation of the *Boston Daily Post* for August was 418,562, a gain of 59,056 copies per day over August in the previous year. What per cent of increase is shown ?

44. In 1912 the population of the United States was approximately 95,000,000. The family expenditures were as follows :

ARTICLE	AMOUNT
Food	\$8000000000
Clothing	3000000000
Intoxicants	2000000000
Tobacco	1000000000
Fuel	800000000
Life Insurance	600000000
Furniture	600000000

(a) What per cent of the total expenditure was spent for tobacco and intoxicants combined?

(b) What is the total expenditure on these items for each person in the United States?

45. A dealer sold a buggy for \$98, an advance of $16\frac{2}{3}\%$ over the cost. If he extends to the purchaser credit for 6 mo. without interest, what was his real per cent of gain, money being worth 6%?

46. An agent sells 80,000 lb. of wheat at 90¢ per bushel. He accepts a 60-day note, which, when discounted at 5%, pays the bill. What is the face of the note?

47. A owes four bills which he pays by New York drafts, the amounts being as follows: \$49.50, \$236.75, \$158.76, and \$61.10. If exchange is $\frac{1}{8}\%$, what exchange does he pay?

48. A buys goods to the amount of \$4586.90, less 3% for cash. He holds B's 30-day acceptance for \$1200, and C's 60-day note for \$2256.30 at 5%. If A discounts the acceptance and the note at 5%, and remits the full amount of his purchase by bank draft, exchange being $\frac{1}{10}\%$, what does he save by paying cash?

49. A man has \$25,050 to invest. Which will yield the more in one year, Norfolk and Western R. R. at 167, no brokerage, paying 5% dividends, or a deposit in a savings bank at 3% interest yearly?

50. A business property is valued at \$200,000. If the owner pays $1\frac{1}{2}\%$ taxes, and allows 10% of the earnings for depreciation of the building, for how much must he rent it to clear 7% on his investment?

APPENDIX.

THE METRIC SYSTEM

The metric system is a decimal system of weights and measures. The United States Government requires the use of the metric system of measures in all medical work of the navy and war departments, and in the public health and marine hospital service.

Linear Measure

The unit of linear measure is the *meter*.

TABLE

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.)
10 meters	= 1 decameter (Dm.)
10 decameters	= 1 hectometer (Hm.)
10 hectometers	= 1 kilometer (Km.)
10 kilometers	= 1 myriameter (Mm.)

Square Measure

The unit of square measure is the *square meter* for small areas, and the *are* of 100 sq. m. for land areas.

TABLE

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.)
100 square centimeters	= 1 square decimeter (sq. dm.)
100 square decimeters	= 1 square meter (sq. m.)
100 square meters	= 1 square decameter (sq. Dm.)
100 square decameters	= 1 square hectometer (sq. Hm.)
100 square hectometers	= 1 square kilometer (sq. Km.)

Land Measure

The unit of land measure is the *are*.

TABLE

100 centares (ca.)	= 1 are (a.) = 100 square meters
100 ares	= 1 hectare (Ha.) = 10000 square meters

Cubic Measure

The unit of volume is the *cubic meter*.

TABLE

1000 cubic millimeters (cu. mm.)	= 1 cubic centimeter (cu. cm.)
1000 cubic centimeters	= 1 cubic decimeter (cu. dm.)
1000 cubic decimeters	= 1 cubic meter (cu. m.)

Wood Measure

The unit of wood measure is the *stere*.

TABLE

10 decisteres (ds.)	= 1 stere (s.) = 1 cubic meter
10 steres	= 1 decastere (Ds.) = 10 cubic meters

Measure of Capacity

The unit of capacity for either solids or liquids is the *liter*, which is equal in volume to 1 cu. dm.

TABLE

10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 liter (l.)
10 liters	= 1 decaliter (Dl.)
10 decaliters	= 1 hectoliter (Hl.)
10 hectoliters	= 1 kiloliter (Kl.)

Measure of Weight

The unit of weight is the *gram*, which is the weight of 1 cu. cm. of distilled water in a vacuum, at its greatest density (39.2° F.). It weighs 15.4324 gr.

TABLE

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 decagram (Dg.)
10 decagrams	= 1 hectogram (Hg.)
10 hectograms	= 1 kilogram (Kg.)
10 kilograms	= 1 myriagram (Mg.)
10 myriagrams	= 1 quintal (Q.)
10 quintals	= 1 tonneau (T.)

TABLES OF EQUIVALENTS**Linear Measure**

1 inch = 2.54 centimeters	1 centimeter = .3937 of an inch
1 foot = .3048 of a meter	1 decimeter = .328 of a foot
1 yard = .9144 of a meter	1 meter = 1.0936 yards
1 rod = 5.029 meters	1 decameter = 1.9884 rods
1 mile = 1.6093 kilometers	1 kilometer = .62137 of a mile

Surface Measure

1 square inch = 6.452 square centimeters
1 square foot = .0929 square meter
1 square yard = .8361 square meter
1 square rod = 25.293 square meters
1 acre = 40.47 ares
1 square mile = 259 hectares
1 square centimeter = .155 square inch
1 square decimeter = .1076 square foot
1 square meter = 1.196 square yards
1 are = 3.954 square rods
1 hectare = 2.471 acres
1 square kilometer = .3861 square mile

Cubic Measure

1 cubic inch	= 16.387 cubic centimeters
1 cubic foot	= 28.317 cubic decimeters
1 cubic yard	= .7646 cubic meter
1 cord	= 3.624 steres
1 cubic centimeter	= .061 cubic inch
1 cubic decimeter	= .0353 cubic foot
1 cubic meter	= 1.308 cubic yards
1 stere	= .2759 cord

Measures of Capacity

1 dry quart	= 1.101 liters	1 liter	= .908 dry quart
1 liquid quart	= .9463 liter	1 liter	= 1.0567 liquid quarts
1 liquid gallon	= .3785 decaliter	1 decaliter	= 2.6417 liquid gallons
1 peck	= .881 decaliter	1 decaliter	= 1.135 pecks
1 bushel	= .3524 hectoliter	1 hectoliter	= 2.8377 bushels

Measures of Weight

1 grain Troy	= .0648 gram
1 ounce Troy	= 31.104 grams
1 ounce avoirdupois	= 28.35 grams
1 pound Troy	= .3732 kilogram
1 pound avoirdupois	= .4536 kilogram
1 ton (short)	= .9072 ton
1 gram	= 15.432 grains Troy
1 gram	= .03215 ounce Troy
1 gram	= .03527 ounce avoirdupois
1 kilogram	= 2.679 pounds Troy
1 kilogram	= 2.2046 pounds avoirdupois
1 ton	= 1.1023 short tons

Convenient Equivalent Values

1 cubic centimeter of water	= 1 milliliter of water, and weighs 1 gram = 15.432 grains
1 cubic decimeter of water	= 1 liter of water, and weighs 1 kilogram = 2.2046 pounds
1 cubic meter of water	= 1 kiloliter of water, and weighs 1 ton = 2204.6 pounds

VALUES OF FOREIGN COINS

COUNTRY	LEGAL STANDARD	MONETARY UNIT	VALUE IN TERMS OF U. S. MONEY
Argentine Republic . . .	Gold	Peso	\$ 0.9647
Austria-Hungary . . .	Gold	Crown	.2030
Belgium	Gold and silver	Franc	.1930
Brazil	Gold	Milreis	.5460
Canada	Gold	Dollar	1.0000
Central American States:			
Costa Rica	Gold	Colon	.4650
British Honduras . . .	Gold	Dollar	1.0000
Nicaragua	Gold	Cordova	1.0000
Guatemala	Silver		
Honduras		Peso	.4340
Salvador			
Chile	Gold	Peso	.3650
China	Silver	Tael	.649 to .723
Denmark	Gold	Crown	.2680
Egypt	Gold	Pound (100 piasters)	4.9430
Finland	Gold	Mark	.1930
France	Gold and silver	Franc	.1930
German Empire	Gold	Mark	.2380
Great Britain	Gold	Pound sterling	4.8665
Greece	Gold and silver	Drachma	.1930
Italy	Gold and silver	Lira	.1930
Japan	Gold	Yen	.4980
Mexico	Gold	Peso	.4980
Netherlands	Gold	Florin	.4020
Norway	Gold	Crown	.2680
Panama	Gold	Balboa	1.0000
Philippine Islands . . .	Gold	Peso	.5000
Portugal	Gold	Escudo	1.0800
Russia	Gold	Ruble	.5150
Spain	Gold and silver	Peseta	.1930
Sweden	Gold	Crown	.2680
Switzerland	Gold	Franc	.1930
Venezuela	Gold	Bolivar	.1930

SQUARE ROOT

The square root of a number is one of the two equal factors of that number.

Find the square root of 16796.16.

$$\begin{array}{r}
 1'67'96.16' \quad \underline{129.6} \\
 1 \\
 22 \overline{) 67} \\
 \underline{44} \\
 249 \overline{) 2396} \\
 \underline{2241} \\
 2586 \overline{) 15516} \\
 \underline{15516}
 \end{array}$$

Point off the number into periods of two places each, beginning at the decimal point. Find, by inspection, the perfect square in 1, the first period, and place it in the root. Square 1, the figure already found, and place the result under 1, the first period. Subtract this square from the first period, and bring down the next period, 67.

Double 1, the root already found, and place the result at the left of 67 as a trial divisor. 2 will be contained in 6, the trial dividend of 67, 3 times. If we place this 3 in the trial divisor and in the root and multiply, the result, 69, cannot be subtracted from 67, so we must use 2 as the second figure in the root. Place 2 in the root and in the trial divisor, and multiply as shown. Write the result, 44, under the 67, subtract, and to the remainder (23) bring down the next period of the number (96). 2396 will be the new dividend.

Double the root already found, divide 24 into 239, put 9 in the root and in the divisor, and multiply. Subtract 2241 from 2396. To the remainder bring down the next period. Double 129, divide 258 into 1551, put 6 in the root and in the divisor, and multiply.

The decimal point is placed after the 9 in the root because the integers have all been used when 9 is obtained as a figure of the root.

NOTE. The square root of a fraction is obtained by extracting the square root of both numerator and denominator; *e.g.*, $\sqrt{\frac{9}{16}} = \frac{3}{4}$. Or, change the fraction to a decimal and extract as above.

ABBREVIATIONS USED IN BUSINESS

A. . . .	acre ; acres	dr. . . .	debit
acc. or %	account	dr. . . .	debtor
alt. or h.	altitude	ea. . . .	each
amt. . .	amount	e.g. . . .	for example
@ . . .	at	etc. . . .	and so forth
av. . . .	average	ex. . . .	example
bal. . .	balance	exch. . .	exchange
bbl. or brl.	barrel ; barrels	f. . . .	franc
bdl. . . .	bundle ; bundles	far. . . .	farthing
bg. . . .	bag ; bags	f. o. b. .	free on board
bl. . . .	bale ; bales	frt. . . .	freight
B/L . . .	bill of lading	ft. or ' .	foot ; feet
bot. . . .	bought	gal. . . .	gallon ; gallons
br't for'd	brought forward	gi. . . .	gill ; gills
bu. . . .	bushel ; bushels	gr. . . .	gross
bx. . . .	box ; boxes	h	hypotenuse
cd. . . .	cord ; cords	hhd. . . .	hogshead
chg. . . .	charge	hr. . . .	hour ; hours
ck. . . .	check	i.e. . . .	that is
c/o . . .	care of	in. or " .	inch ; inches
co. . . .	company	ins. . . .	insurance
c. o. d. .	collect on delivery	inst. . . .	present month
coll. . .	collection	int. . . .	interest
com. . . .	commission	K or k . .	area
cr. . . .	credit ; creditor	£	pound sterling
cs. . . .	case ; cases	lb. . . .	pound ; pounds
ct. or ¢ .	cent ; cents	L. C. D. .	least common denominator
ctg. . . .	cartage	L. C. M. .	least common multiple
cu. . . .	cubic	ltd. . . .	limited
cwt. . . .	hundredweight	M. . . .	mark
d. . . .	pence	m. . . .	mill ; mills
da. . . .	day ; days	mdse. . .	merchandise
dft. . . .	draft	mi. . . .	mile ; miles
disc. . .	discount	min. or ' .	minute ; minutes
doz. . . .	dozen	mo. . . .	month ; months

mortg. . . mortgage
no. or # . . . number
o. k. . . all correct
oz. . . ounce ; ounces
p. . . page
pay't . . . payment
pc. . . piece
pd. . . paid
% . . . per cent
pfd. . . preferred
pkg. . . package
pp. . . pages
pr. . . pair ; pairs
prox. . . next month
pt. . . pint ; pints
pwt. . . pennyweight
qr. . . quire ; quires
qt. . . quart ; quarts
rd. . . rod ; rods
rec'd . . . received

rec't . . . receipt
ren'd . . . rendered
rm. . . ream
ry. . . railway
s. . . shilling ; shillings
sec. or '' . . . second ; seconds
sec. . . section ; sections
sq. . . square
st. ht. . . slant height
T. . . ton ; tons
twp. . . township
ult. . . last month
via . . . by way of
viz. . . namely
vol. . . volume
wk. . . week ; weeks
wt. . . weight
yd. . . yard ; yards
yr. . . year ; years

INDEX

(The figures refer to pages.)

- A. B. A. checks, 204
- Abbreviations, 243
- Above par, 192
- Acceptance, 206
- Account, checking, 158
 - purchase 134
 - sales, 134, 136
 - savings, 159
- Acre, 68
- Addition, 7
 - checking, 12
 - denominate numbers, 73
 - fractions, 51
 - horizontal, 14
- Ad valorem duty, 183
- Agate, 72
- Aliquot parts, 47
- Altitude, 93, 97
- Amount, 117, 144
- Angle, 92
- Annuity, 155
- Anticipation, 125
- Apothecaries' fluid measure, 70
 - weight, 66
- Apportionment, 181
- Approximate measures, 107
- Approximations, 107
- Arbitrage, 211
- Area, 93
 - of circle, 95
 - of parallelogram, 94
 - of triangle, 94
- Assessment, stock, 192
 - tax, 180
- Assessors, 180
- Avoirdupois weight, 64
- Balance, 158
- Balancing account, 20
- Bale, 72
- Bank, 157
 - check, 157, 161, 203
 - discount, 164
 - draft, 203
 - Federal reserve, 158
 - National, 157
 - note, 64, 157
 - private, 158
 - savings, 158
 - state, 158
- Barrel, 66, 70
- Base, line, 68, 117
 - in mensuration, 93
 - in percentage, 117
- Below par, 192
- Beneficiary, 189
- Bill, clean, 210
 - documentary, 210
 - of exchange, 210
- Board foot, 104
- Bond, 196
 - coupon, 196
 - registered, 198
 - table, 198
- Bricks, 105
- Broker commission, 133
- Brokerage, 133, 193
- Building and Loan Associations, 164
- Bundle, 72
- Bushel, 70
- Cable transfer, 209
- Canadian money, 64
- Cancellation, 42
 - interest method, 146
- Capital, 213
 - loans, 163
- Carat, 66
- Carload lots, 215
- Carpeting, 100
- Cash discount, 124
- Cashier's check, 204
- Cask, 66
- Casting out 9's, 12, 24, 29
- Cent, 63
- Cental, 66
- Centime, 64
- Century, 71
- Certificate, coupon, 196
 - of deposit, 160
 - stock, 193
- Certified check, 203
- Chain, 68
- Check, A.B.A., 204
 - accuracy, 12, 18, 24, 29
 - bank, 161, 203
 - cashier's, 204
 - certified, 203
 - travelers', 204
- Checking account, 158

- Circle, 71, 93
- Circular measure, 71
- Circumference, 93
- Class rates, 215
- Clean bills, 210
- Clearing house, 209
- Closed policy, 185
- Coins, Canada, 64
- United States, 63
- Coinsurance, 187
- Collateral, 162
- notes, 162
- Collection charge, 203
- Collector, 133
- Combinations in addition, 7
- Commercial, discount, 123
- draft, 204
- fractions, 55
- time table, 71
- weight, 64
- Commission, 133, 193
- merchant, 133
- Commodity rates, 215
- Common, divisor, 40
- fraction, 43
- interest, 144
- multiple, 41
- stock, 193
- Complement, 18
- Compound interest, 153
- subtraction, 106
- time, 106
- Cone, 97
- Consignee, 134
- Consignment, 134
- Consignor, 134
- Cord, 69
- Corporation, 192
- tax, 180
- Cost, first or prime, 130
- gross, 120
- net, 120
- Counting table, 72
- Coupon, bond, 196
- certificate, 196
- Credit, 157
- Cube, 39, 96
- Cubic measure, 69
- Customs, 183
- Cylinder, 97
-
- Date of maturity, 164
- Day, 71
- of grace, 164
- Decimal, 43
- point, 60
- Decime, 64
- Degree, 71
- Denominate numbers, 63
- Denomination, 198
-
- Denominator, 43
- least common, 46
- Deposit, 158
- certificate of, 203
- slip, 158
- Depositor's ledger, 20
- Diagonal, 93
- Diameter, 93
- Diamond weight, 66
- Difference, in time, 106
- in percentage, 117
- Discount, bank, 162, 164
- commercial, 123
- series, 127
- time, 164
- true, 168
- Divisibility tests, 38
- Division, 28
- Divisor, greatest common, 40
- in fractions, 58
- Documentary bills, 210
- Dollar, 63
- Dozen, 72
- Draft, acceptance, 206
- bank, 203
- commercial, 205
- sight, 206
- time, 206
- Dram, 65, 67
- Drawee, 205
- Drawer, 205
- Dry measure, 70
- Duty, 183
-
- Eagle, 63
- Endowment policy, 189
- English money, 64
- Equalizing investment, 214
- Equation, 88
- Even number, 38
- Exact interest, 151
- time table, 166
- Exchange, 201
- bill of, 210
- cost of, 209
- domestic, 201
- foreign, 201, 209
- indirect, 210
- par of, 210, 211
- rate of, 201, 211, 212
- Exponent, 39
- Express money order, 202
-
- Factor, 38
- method, 44
- Factoring, 38
- Farthing, 64
- Fathom, 67
- Federal reserve bank, 158
- Fee, 202

- Fire insurance, 185
Firkin, 66
Flooring, 103
Foot, 67
 board, 104
Footing, 14
Formulas, 94, 98, 118, 121
Fractions, 43
Franc, 64
Free list, 183
Freight, rates, 215
 tariff, 215
French money, 64
Fund, 157
 sinking, 155
Furlong, 67

Gain and Loss, 120
Gallon, 70
G. C. D. method, 44
Geographical mile, 67
German money, 64
Gill, 70
Gold, certificates, 64
 coins U. S., 63
 fineness of, 66
Government, land measure, 68
 money, 63
Grain measure, 66, 67
Graph, 79
Greatest common divisor, 40
Great gross, 72
Gross, 72
 cost, 120, 134
 proceeds, 134
 weight, 66
Guaranty, 134
Gunter's chain, 68

Hand, 67
Heaped bushel, 70
Horizontal addition, 14
Hour, 71
Hundredweight, 65
Hypotenuse, 92

Import duties, 183
Improper fraction, 45
Inch, 67
Income tax, 180
Incorporated companies, 192
Indemnify, 185
Indirect exchange, 210
Indirect tax, 180, 183
Indorsement, 161
Industrial loans, 163
Inheritance tax, 180
Insolvency, 213
Insurance, 185
 fire, 185
 life, 189
 table of, 190
Interchange of fractions, 43
Interest, 144
 bearing debts, 144
 cancellation method, 145
 common, 144
 compound, 153
 exact, 151
 formulas, 144
 laws, 144
 6 per cent method, 148
 60-day method, 147
 special methods, 151
Investment loans, 163
Invoice, date of, 124

Joint rate commission, 215
Judgment notes, 162

Karat, 66
Keg, 66
Knot, 67

Land measure, 68
League, 67
Leakage, 183
Least common denominator, 46
Least common multiple, 41
Legal rate of interest, 144
Letter of credit, 210
Liabilities, 122, 213
License fee, 180
Line, 92
Linear measure, 67
Link, 68
Liquid measure, 70
Load, 69
Loans, 162
London exchange, 202
Long measure, 67
Long-time loans, 163
Long ton, 65
Loss, 213
Lumber 104

Making change, 18
Manual training, 107
Mark, 64
Market value, 192
Marking goods, 128
Maturity, 144
Measures, of capacity, 64
 of value, 63
 of weight, 64
Mensuration, 92
Merchants' rule, 171
Meridian, principal, 68
Metric system, 237
Mile, 67, 68

- Mill, 63
- Minimum weight, 216
- Minute, circular measure, 71
 - time measure, 71
- Miscellaneous, measures, 72
 - problems, 30, 83, 109, 137, 172, 221
- Mixed number, 45
- Money, centers, 202
 - tables, 63
- Month, 71
- Mortgage loans, 163
- Multiplication, 23
 - of commercial fractions, 55
 - of fractions, 53
 - rapid method of, 25
- National Bank, 157
- Negotiable paper, 162
- Net cost, 120
 - gain, 213
 - loss, 213
 - price, 124
 - proceeds, 134
 - selling price, 121
 - weight, 66
 - wholesale price, 124
- New York exchange, 202
- Nonnegotiable paper, 162
- Note, collateral, 162
 - judgment, 162
- Numerator, 43
- Open policy, 185
- Ounce, apothecaries', 67
 - commercial, 65
 - Troy, 66
- Painting, 100
- Papering, 100
- Par, 192
 - of exchange, 210
 - value, 192
- Parallel lines, 92
- Parallelogram, 93
- Parcel post, 219
- Parenthesis, 18
- Partial payments, 169
- Partnership, 213
- Pass book, 158
- Peck, 70
- Pence, 64
- Pennyweight, 66
- Percentage, 117
- Perch, 69
- Perimeter, 93
- Perpendicular, 92
- Pfennig, 64
- Pica, 72
- Pint, 70
- Plastering, 100
- Point, 72
- Policy, 185
 - endowment, 189
 - open, 185
 - term, 189
 - valued, 185
 - whole life, 189
- Poll tax, 182
- Polygon, 92
- Postal, money order, 202
 - savings bank, 162
 - savings certificate, 162
- Pound, commercial, 65
 - Troy, 66
 - sterling, 64
- Power, 39
- Practical measurements, 100
- Preferred, risk, 187
 - stock, 193
- Premium, insurance, 185
 - stock, 192
- Present worth, 168, 213
- Price, list, 124
 - net, 124
- Prime cost, 134
 - factor, 38
 - number, 38
- Principal, 134, 144
 - meridian, 68
- Printers' measure, 72
- Prism, 96
- Private bank, 158
- Proceeds, bank, 164
 - gross, 134
 - net, 134
- Profit, 130, 213
- Proper fraction, 45
- Properties of numbers, 38
- Property tax, 180
- Protest, 206
- Pyramid, 97
- Quadrilateral, 92
- Quart, 70
- Quintal, 66
- Quire, 72
- Radical sign, 40
- Radius, 93
- Railroad, 215
- Range, 68
- Rate, carload, 215
 - class, 215
 - commodity, 215
 - exchange, 201
 - legal interest, 144
 - per cent, 117, 144
 - railroad, 215
- Ream, 72

- Reciprocal, 58
Rectangle, 93
Reduction, of denominate numbers, 73
 of fractions, 44
Registered bond, 198
Resources, 122, 213
Retail merchants' table, 131
Review problems, 227
Right angle, 92
 triangle, 92
Road taxes, 180
Rod, 67, 68
Roofing, 103
Root, square, 242

Savings bank; 158, 162
Score, 72
Scruple, 67
Second, circular measure, 71
 time measure, 71
Section of land, 69
Security, 162
Share, 192
Sheet, 72
Shilling, 64
Shipment, 134
Short rate, 187
Short-time loans, 163
Sight draft, 206
Simple interest, 144
 table, 149
Sinking fund, 155
Six per cent method, 148
Sixty-day method, 147
Size, 67
Slant height, 97
Solar year, 71
Solids, 96
Sovereign, 64
Special interest methods, 151
Specific duty, 183
Square, 68, 93
 measure, 68
 roofing, 103
 root, 40, 242
Standard time, 71
State bank, 158
Statute mile, 67
Stocks, 192
 assessment, 192
 broker, 193
 certificate, 192
 common, 193
 companies, 192
 preferred, 193
Stone, 105
Stub of check, 161
Subsidiary coins, 64
Substitution, 89
Subtraction, 17

checking, 18
compound, 106
fractions, 52
Surface measure, 68
Surveyors' long measure, 68

Tables, aliquot parts, 47
 bond, 198
 commercial, 65
 compound interest, 154
 counting, 72
 denominate numbers, 63
 domestic exchange, 202
 foreign coins, 241
 foreign exchange, 211, 212
 import duties, 183
 insurance, 190
 metric system, 237
 miscellaneous measures, 72
 money, 63
 multiplication, 23
 paper, 72
 parcel post, 219
 percentage, 47
 printers', 72
 retail merchants', 131
 simple interest, 149
 sinking fund, 155
 time, 166
Tare, 66, 183
Tariff, 180
 freight, 215
Tax, 180
 apportionment, 181
 assessment, 180
 direct, 180
 formulas, 180
 indirect, 180, 183
 transfer, 200
Telegraphic money order, 202
Teller, 158
Term, fraction, 43
 of credit, 124
 policy, 189
Tests of divisibility, 38
Time, 144
 discount, 164
 draft, 206
 measure, 71
 standard, 72
 table, 166
Ton, 65
Township, 69
Trade discount, 123
Transfer, of money, 201
 tax, 200
Travelers' checks, 204
Trial balance, 35
Triangle, 92
Troy weight, 66

True discount, 168, 192, 194
Trust companies, 158
Two-figure combinations, 7

United States, customs, 183
 money, 63
 rule, 170
Usury, 144

Value, of foreign coins, 241
 insurable, 186
 market, 192
 par, 192
Valued policy, 185
Vinculum, 18
Volume, 97

Week, 71
Weight, apothecaries', 66
 commercial, 64
 Troy, 66
Whole-life policy, 189
Wholesale and retail profits, 130
Winchester bushel, 70
Withdrawals, 160
With exchange, 205
Without exchange, 205
Wood, 105

Yard, 67
Year, 71

Zone, 220

Engerie Koch
VS

Squires Inc.

Eugene Koch